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Dynamic Analysis of a Clamped-Clamped Pipe Conveying Single-Phase Fluid

Valéria França[1], Jian Su[2]

[1]valeria.franca@coppe.ufrj.br, Departamento de Engenharia Nuclear/COPPE
Universidade Federal do Rio de Janeiro
21941-972, Rio de Janeiro, RJ
[2]sujian@nuclear.ufrj.br, Programa de Engenharia Nuclear/COPPE
Universidade Federal do Rio de Janeiro
21941-972, Rio de Janeiro, RJ

1. Introduction

Flow-induced vibrations (FIV) are a well known problem in Nuclear Engineering, since reactors and components that compose nuclear power plants are susceptible to this phenomenon [1,2]. The fluid-structure interaction (FSI), present in several systems that integrate the plant, can generate undesirable vibrations and affect the plant operation.

Pipes are essential components in nuclear industry and are mainly responsible for fluids transport. Therefore, they are subject to flows and may suffer excessive vibrations, which could cause operational failures, resulting in a potential risk to safety, besides economic losses.

The objective of this work is to develop a mathematical tool for the dynamic analysis of a pipe conveying a single-phase fluid. The method used to solve the problem governing equations was the Generalized Integral Transform Technique (GITT), which is an analytical-numerical methodology that transforms partial differential equations (PDEs) into an ordinary differential equations (ODEs), bringing the advantages of a hybrid solution to problems involving fluid flow and heat conduction.

2. Methodology

Consider a clamped-clamped pipe conveying a single-phase fluid, modeled as an Euler-Bernoulli beam [1,2]. Fig. 1 represents the physical model geometry in which a pipe of length L, cross-sectional area A, mass per unit length m, bending stiffness EI, transportS a fluid of mass per unit length M and velocity U.

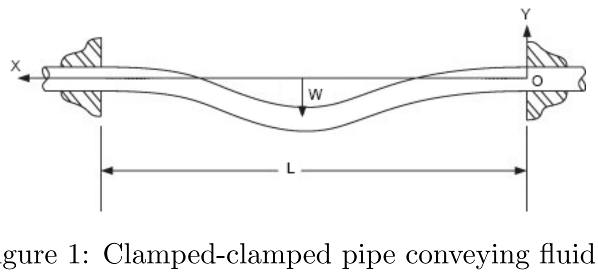


Figure 1: Clamped-clamped pipe conveying fluid[1].

The fluid-structure interaction is represented by a set of governing equations that couple fluid dynamics and structural mechanics, and in this problem can be represented by the following dimensionless equation for small lateral displacements,

$$\frac{\partial^4 w}{\partial x^4} + u^2 \frac{\partial^2 w}{\partial x^2} + 2\sqrt{\beta} u \frac{\partial^2 w}{\partial x \partial t} + \frac{\partial^2 w}{\partial t^2} = 0, \tag{1}$$

where w is the dimensionless lateral deflection of the pipe, β is the mass ratio, x is the dimensionless axial coordinate, u the dimensionless flow velocity.

The dimensionless boundary conditions are given as follows:

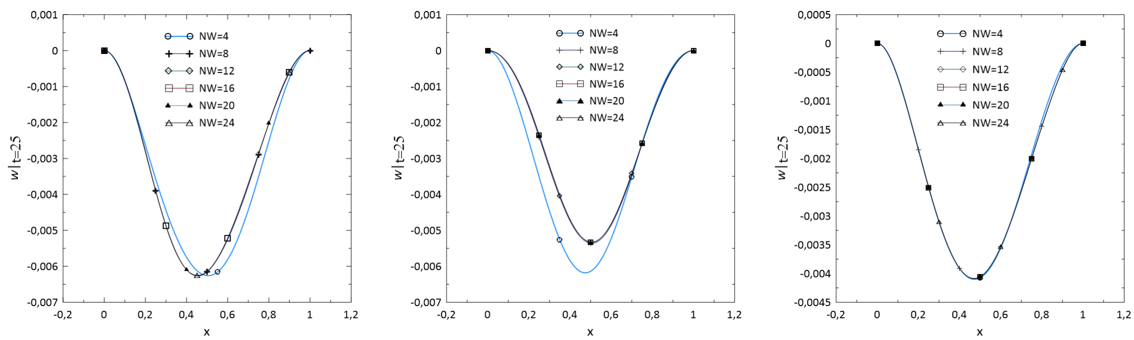
$$w(0,t) = 0, \quad w(1,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad \frac{\partial w(1,t)}{\partial x} = 0. \tag{2}$$

The Generalized Integral Transform Technique (GITT) was applied to solve the equations. This approach is based on the creation of a pair of equations that represent the transformation and inversion of the original problem, using the series expansion obtained from an associated Sturm-Liouville problem.

3. Results and Discussion

The convergence analysis of the integral transform solution was performed using GITT for different truncation orders, $NW = 4$, 8, 12, 16, 20 and 24, analyzing the aspects of displacement and time history.

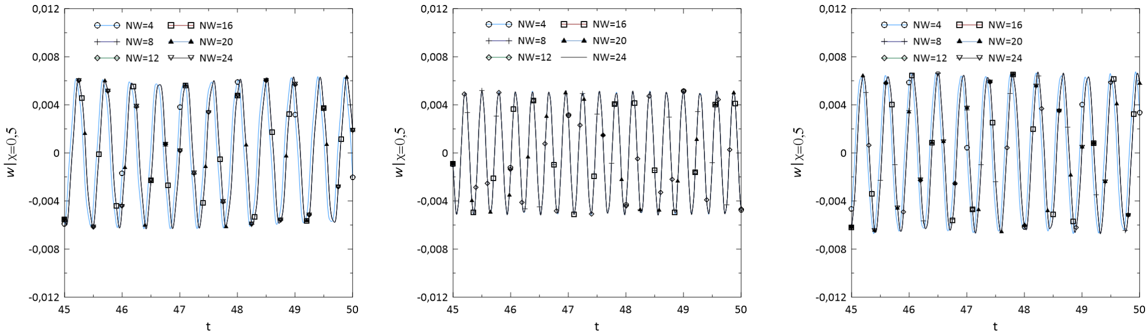
The dimensionless transverse displacement $w(x,t)$ convergence was analyzed at different positions of the pipe conveying a fluid, and it was examined from the increase of the truncation terms NW at the dimensionless times t= 25. Fig. 2 illustrates the displacement profile for different combinations of u and β, showing that for convergence with three significant digits it is necessary a truncation order $NW \leq 12$.



(a) (b) (c)

Figure 2: Dimensionless transverse deflection convergence analyses of $w(x,t)$ for $t = 25$ with different truncation orders NW for (a) $u = 4.5$ and $\beta = 1.0$, (b) $u = 4.5$ and $\beta = 0.5$ and (c) $u = 1.5$ and $\beta = 0.5$.

Similarly, the temporal evolution of the transverse displacement at the center point of the pipe is shown in Fig. 5. Considering the time interval, $t \in [45{,}50]$, and also varying β and u, it is noticed that the convergence is quite favorable.

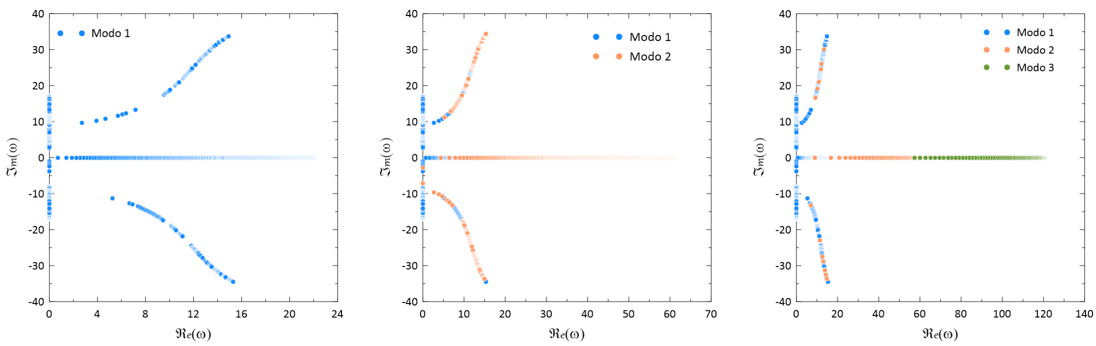


(a) (b) (c)

Figure 3: Time history transverse displacement convergence analyses at the center point of the pipe $w(0.5, t)$ with different truncation orders NW para (a) $u = 4.5$ and $\beta = 1.0$, (b) $u = 4.5$ and $\beta = 0.5$ and (c) $u = 1.5$ and $\beta = 0.5$.

Stability was analyzed by varying the dimensionless velocity, u, considering different mass ratios β=0.1, 0.5 and 1.0 to obtain the eigenfrequencies. The behavior of the first three modes of dimensionless frequency referring to the displacement of the pipe to $\beta = 0.1$ is illustrated in Fig. 4. According to Fig.4(a), the eigenfrequencies are purely real and with increasing velocity, the first frequency modes tend to decrease and disappear when $u = \pi$, that's called first critical velocity, u_c, of the fluid. However, when $u > u_c$ the first modes are grouped on the axis Im, becoming the eigenfrequencies purely imaginary, meaning loss of system' stability[2].

The same analysis was performed for the fundamental and second frequency modes, and it can be seen in Fig. 4(b) that the second mode follows the same behavior as the first, but the frequency of the second mode disappears when $u = 2\pi$, called the second critical system velocity. However, in $u > u_c$ the first two modes merge and indicate the beginning of a coupled vibration.

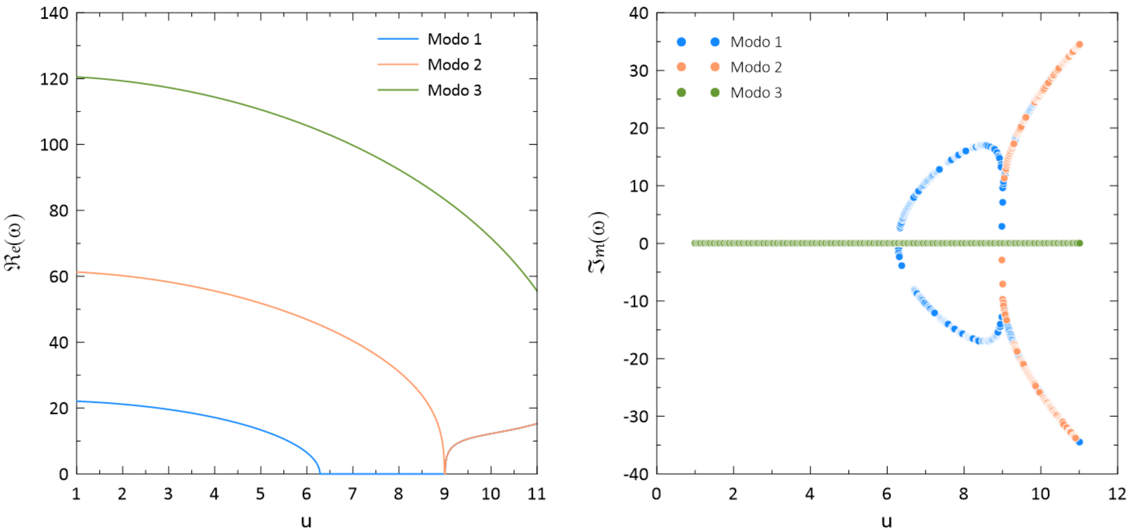


(a) (b) (c)

Figure 4: Dimensionless complex frequency diagrams for a clamped-clamped pipe;(a) $\beta = 0.1$, (b) $\beta = 0.1$ and (c) $\beta = 0.1$.

Modes 1-3 are illustrated in Fig.4(c), in which the system's eigenfrequencies are purely real and decrease to $u = 2\pi$ and after that the values become purely imaginary, characterizing the loss of system stability by divergence through a pitchfork bifurcation.

Fig. 5 illustrates the real and imaginary part of the dimensionless eigenfrequencies for modes 1-3 as velocity changes, showing that eigenfrequencies decrease with increase of u and the system loses stability due to divergence. Also, according to Fig. 2 the system's eigenfrequencies are purely real up to $u_c = 2\pi$ and after this velocity value, it becomes purely imaginary as reported on the literature[2].



(a) (b)

Figure 5: Real and imaginary components of the dimensionless frequency, ω, as functions of the dimensionless flow velocity, u, for $\beta = 0.1$.

4. Conclusions

The application of the generalized integral transform technique (GITT) to obtaining the hybrid solution of the dynamic response of a pipe conveying single-phase fluid has shown to be a good approach to solving the physical problem proposed, presenting accurate results. Also, the stability criterion studied demonstrated the flow velocity influence on the frequencies modes, showing compatibility with the literature.

Acknowledgments

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References

[1] Jijun Gu, "Integral transform solutions of dynamic response of a clamped–clamped pipe conveying fluid", *Nuclear Engineering and Design*, vol. 254, pp. 237–245 (2013).

[2] Paidoussis, M. P.,, *Fluid-structure interactions: slender structures and axial flow*, Wiley Online Library, New York & USA (2007).