



## Simulation of Transients in a Sub-Critical Heterogeneous System in Three-Dimensional Geometry

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### 1. Introduction

Accelerator-Driven System, ADS [1], belong to the new generation of advanced reactors being developed that promise to drastically reduce the life of radioactive waste by, for example, the transmutation process. Subcritical system reactor designs of the ADS type have attracted worldwide attention and are the subject of research and development in several countries. The concept of ADS (frequently called hybrid systems) combines a particle accelerator with a sub-critical core. Most proposals assume proton accelerators, delivering continuous-wave beams. The accelerator is either a linear accelerator (linac) or a circular accelerator (cyclotron). The protons are injected onto a spallation target to produce source neutrons for driving the subcritical core. The target is made of heavy metal in solid or liquid state. Spallation reactions in the target emit a few tens of neutrons per incident proton, which are introduced into the sub-critical core to induce further nuclear reactions. Except for the sub-critical state, the core is very similar to that of a critical reactor. It can be designed to operate either with a thermal or fast neutron spectrum. The purpose of this work is to simulate transients associated with heterogeneous ADS subcritical system in three-dimensional geometry. It adopted the neutron diffusion model that leads the spatial kinetics equations. These equations are solved by the known numerical method of finite differences. The simulations are performed considering transients related to the variations in the intensity of the proton flux provided by the particle accelerator acting in a sub-critical reactor in three-dimensional geometry for two energy groups and six groups of delayed neutron precursors.

### 2. Methodology

The three-dimensional spatial kinetic neutron diffusion equations, for two energy groups, six delayed neutron precursor groups and with the presence of an external source are written as follows:

$$\frac{1}{v_g} \frac{\partial \phi_g(\mathbf{r}, t)}{\partial t} - D_g \nabla^2 \phi_g(\mathbf{r}, t) + \Sigma_{Rg} \phi_g(\mathbf{r}, t) = \chi_{P,g} (1 - \beta) \sum_{g'=1}^2 \nu \Sigma_{fg'} \phi_{g'}(\mathbf{r}, t) + \sum_{\substack{g'=1 \\ g' \neq g}}^2 \Sigma_{sgg'} \phi_{g'}(\mathbf{r}, t) + \sum_{l=1}^6 \chi_{D,l} \lambda_l C_l(\mathbf{r}, t) + S_{es,g}(\mathbf{r}, t), \quad g = 1, 2 \quad (1)$$

$$\frac{\partial C_l(\mathbf{r}, t)}{\partial t} = -\lambda_l C_l(\mathbf{r}, t) + \beta_l \sum_{g'=1}^2 \nu \Sigma_{fg'} \phi_{g'}(\mathbf{r}, t) \quad (2)$$

where  $\phi_g(\mathbf{r}, t)$  is the neutron flux,  $D_g$  the diffusion coefficient,  $\Sigma_{Rg}$  the removal cross section,  $\nu \Sigma_{fg}$  the average number of neutrons emitted by fission multiplied by fission cross section,  $\Sigma_{sgg'}$  the scattering cross section,  $S_{es,g}(\mathbf{r}, t)$  the external source (source of spallation), defined in the group  $g$  of energy,  $C_l(\mathbf{r}, t)$  the delayed neutron precursor concentration in precursor group  $l$ , all defined at point  $\mathbf{r}$  and time  $t$ ,  $v_g$  the velocity,  $\chi_{P,g}$  the fission spectrum for prompt neutrons, both in group  $g$ ,  $\chi_{D,l}$  the fission spectrum for delayed neutrons,  $\lambda_l$  the decay constant,  $\beta_l$  the fraction of all fission neutrons emitted per fission, defined in the  $l$  precursor group and finally  $\beta$  the total fraction of fission neutrons which are delayed. Eq. 1 and Eq. 2 are discretized in space and time. The spatial discretization scheme adopted is based on classical formulation of finite differences, implemented in the box schema and to solve time dependent equation system, the analytical integration procedure has been adopted for the precursor concentration equation, Eq. 2, whereas the Methods Implicit Euler is considered for the neutron flux in Eq. 1. A computational code of the numerical implementation was programmed in Fortran language to simulate the transients. A more detailed description of the methodology can be found in [2]. To model a heterogeneous ADS subcritical system, a modified version of the problem originally introduced in [3] was considered representing a simplified three-dimensional PWR model with 77 fuel elements. Two energy groups, six delayed neutron precursor groups, and quarter-nucleus symmetry are considered. The core composition and kinetic data are presented in [2], where, in column five, the mean number of neutrons emitted by the macroscopic fission cross section for each energy group are already multiplied. Some of the nuclear parameters were modified in relation to the values of the original problem, to obtain a  $k_{eff} = 0.95026393$  and a diffusion length equal to 4.032 cm. The core has a height of 200 cm, and its radial geometry can be seen in Fig. 1. The finite difference spatial discretization adopted was  $\Delta x = \Delta y = \Delta z = 4$  cm, with fifty radial planes, each plane with 759 nodes, totaling 37,950 nodes. The initial neutrons flux is given by the flux of the stationary calculation without an external source and the initial precursor concentration is given by Eq. 2 considering the null time derivative and the flux boundary condition was used null.

### 3. Results and Discussion

Two types of transients associated with an ADS reactor will be simulated and will focus on the proton accelerator perturbations, causing variations in the intensity of the proton beam and consequently the intensity of the external source of neutrons. The first transient corresponds to the accelerator beam interruption (ABI) for a short period of time and the second transient to be addressed describes the occurrence of an accelerator beam over-power (ABO) [4]. An external neutron source, located in the center of the reactor core, with a constant intensity equal to  $10^{14}$  neutrons/s was adopted. In ABI transient the reactor is operating critically, and the proton beam of the accelerator is interrupted in the instant in 1s and after 2s over the beam is reconnected. In ABO transient the reactor is operating critically and the intensity of the proton beam of the accelerator is increased by 100% instantaneously and after 2s over the beam has its intensity restored to the initial level. Figs. 2a and 2b show, respectively, the thermal neutron flux at the instant in 3s, at the end of the ABI and ABO. The graphs are for the fluxes located in plane 25, near the mid-height of the core.

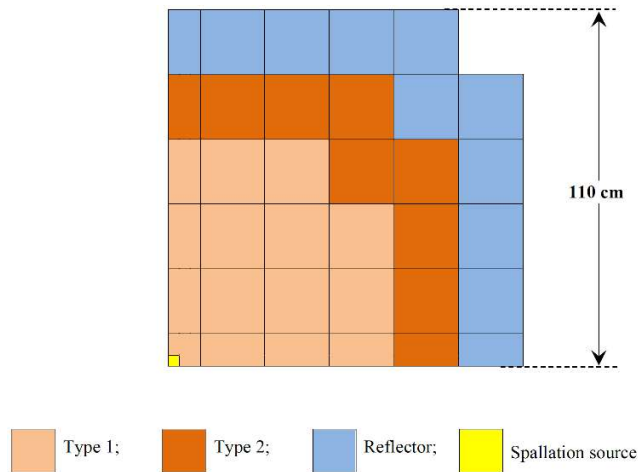


Figure 1: Simplified PWR core radial geometry with spallation source.

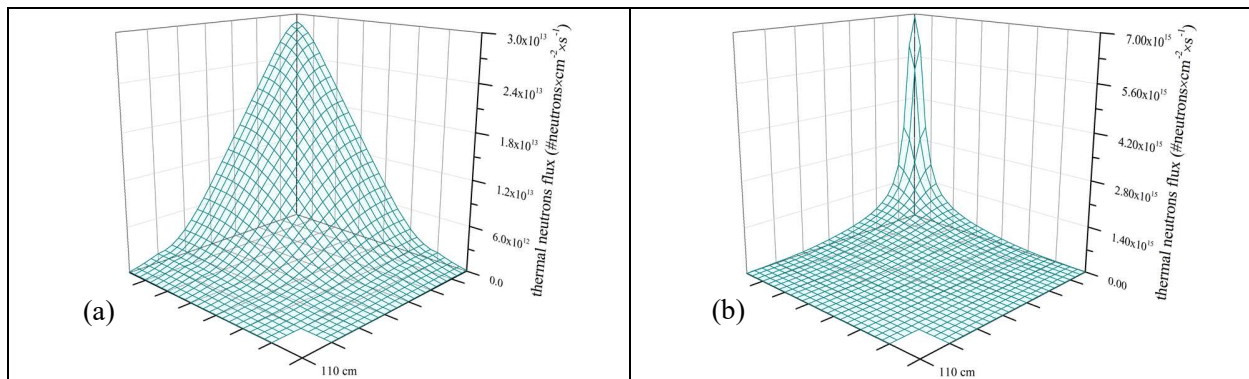


Figure 2: Thermal neutron flux in a plane near the mid-height of the core at the instant in 3s: (a) ABI transient; (b) ABO transient.

In Figs. 3a and 3b the variation of the relative power during a transient of 10s is shown. Relative power is calculated by making the ratio between the power values during the transient with the power at the initial instant. To obtain the power, an arbitrary value for the normalization constant was considered. Fig. 3a shows the behavior of the relative power, considering the simulation in ABI transient. With the interruption of the proton beam in the instant in 1s, an abrupt change in relative power is observed, with an 87% reduction in the power level in a short time interval of 0.025s, there is also an abrupt change in the level of power in the instant in 3s when the accelerator is turned back on. Furthermore, it is observed that between the instants of 1 and 3s, during which the accelerator is off, the relative power is slowly reduced by 35%. This is due to the subcriticality of the ADS system reactor. The behavior of the relative power, considering the simulation in ABI transient can be verified in Fig. 3b. With the increase in the intensity of the accelerator's proton beam at the instant in 1 s, an instantaneous variation of the relative power is observed, with an 87.5% increase in the power level in a short time interval of 0.025 s. With the accelerator operating at normal intensity, the ADS reactor operates at criticality and, therefore, with said increase in beam intensity, the reactor starts to operate “supercritical”. Thus, a gradual increase of 2.35% in relative power can be observed between 1 and 3 s, an interval during which the accelerator is generating a beam of protons with a double intensity. Although there are no other results in the literature to make a comparison, the results obtained are very similar to those obtained in simulations using a slab-type reactor and in two-dimensional geometry [4].

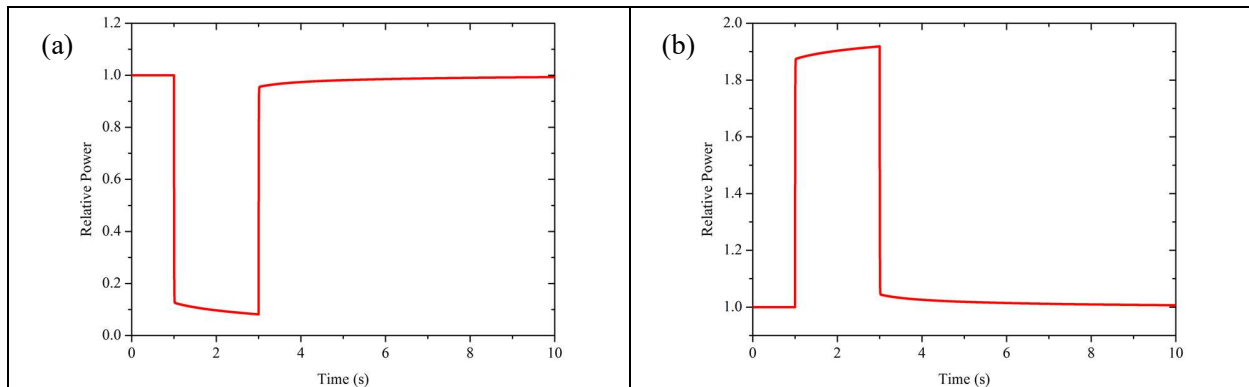


Figure 3: Variation of relative power as a function of time: (a) ABI transient; (b) ABO transient.

#### 4. Conclusions

In this work, the spatial kinetics equations were modeled numerically to obtain the neutron flux distribution and the power of a three-dimensional nuclear reactor. The delayed neutron precursor equation was integrated analytically and the numerical method to discretize spatial kinetics equations was that of finite differences. The dependence in time was solved by the well-known implicit Euler method. A three-dimensional heterogeneous subcritical ADS system with two energy groups, six groups of delayed neutron precursors was modeled. Two transients associated with variations in neutron source intensity were simulated: accelerator beam interruption (ABI) and accelerator beam over-power (ABO). The results obtained are in accordance with expected, when compared with results from other works that used reactors with simpler geometry (1D and 2D). In the future, the developed computational code should be applied to ADS system benchmark problems, considering a more realistic 3D composition and geometry of a core.

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#### References

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