*2021 International Nuclear Atlantic Conference – INAC 2021*
*Virtual meeting, Brazil, November 29 – December 2, 2021*

ISBN: 978-65-594-1256-3

# Comparison of Uncertainty Propagation Methods in the Scope of Probabilistic Risk Assessment

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## 1. Introduction

Probabilistic risk assessment (PRA) is increasingly being used within licensing processes of both nuclear power plants and research reactors. PRAs traditionally uses systematic tools to assess probabilities of occurrence of undesired events and severity of accident scenarios, such as fault trees and event trees. Uncertainty propagation in fault trees is a key step in PRA, because basic events often have wide confidence bounds and they should be treated as random variables to obtain the probabilities of occurrence of top events. This work presents the comparison of two methods of uncertainty propagation in fault trees, their advantages and disadvantages using different types of probability density functions.

## 2. Methodology

Mathematically, risk from an undesired event can be expressed as a product of probability of occurrence of the accident scenarios and magnitude of their consequences, as shown in Eq. 1 [1]:

$$Risk = probability\ of\ occurrence \times magnitude\ of\ consequences \qquad (1)$$

PRA involves the combination and integration of the probabilities (or frequencies) and the magnitude of consequences (severities) for the identified hazards, taking into account the effectiveness of any existing controls and barriers (defence-in-depth levels). It provides an input to risk evaluation and decisions about risk treatment and risk management strategies, though the adoption of adequate risk acceptance criteria [2]. Fig. 1 illustrates an event tree for an initiating event (frequency of occurrence $\lambda$), taking into account the failure probabilities of defence-in-depth levels ($P_1$, $P_2$), leading to different accident scenarios ($S_1$, $S_2$, $S_3$, $S_4$). This tool is usually used in PRAs together with fault trees to estimate low probability events, such as failure probability of protection systems of nuclear reactors. Fig. 2 illustrates a fault tree to estimate the probability distribution of top event, T (e.g., failure of a defence-in-depth level) when the probability distributions of the basic events (X1, X2, X3, … X7) are known [3]. This kind of uncertainty propagation in fault trees plays an important role in licensing processes of nuclear facilities, mainly to evaluate if the plants meet the risk acceptance criteria.

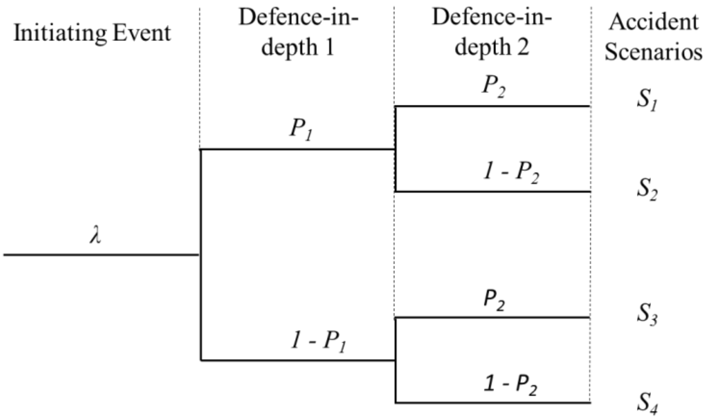


Figure 1: Example of an event tree for an initiating event (frequency of occurrence $\lambda$), taking into account the failure probabilities of defence-in-depth levels and leading to different accident scenarios.

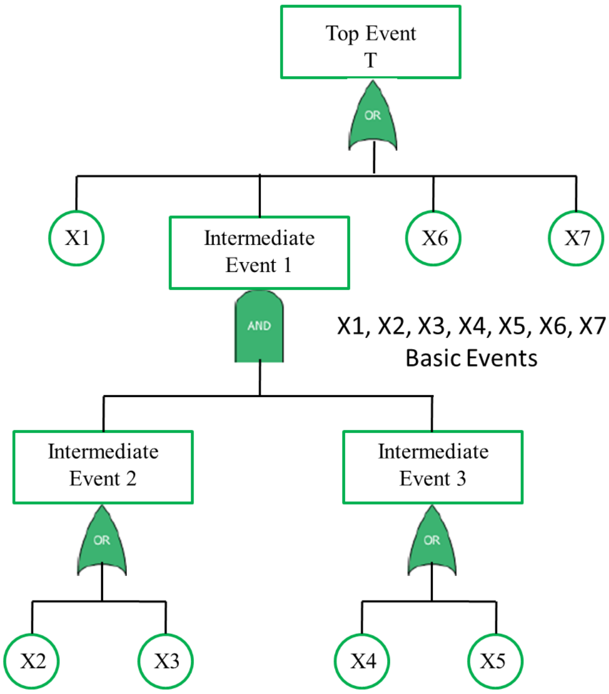


Figure 2: Example of a fault tree for estimating the probability distribution of top event from the probability distributions of basic events, combined by "OR" and "AND" gates.

Two classical methods of uncertainty propagations in fault trees were chosen for study: Monte Carlo simulation and method of moments.

Monte Carlo simulation (MCS) is a statistical method for modeling based on direct simulation of systems or processes. With random sampling of probability density functions of input parameters, the uncertainties of output parameters are obtained. It is simple in principle, but requires the use of computer programs to be implemented because the high number of samples necessary to get accurate results. This work uses the BlockSim and RENO software, developed by Reliasoft® Corporation, for building and running complex event analyses for implementing uncertainty propagation in fault trees using MCS [4].

The method of moments [5] assumes that the basic events are independent to propagate the uncertainties in fault trees, using the coefficient of variations, C and C´, given by Eq. 2 and Eq. 3, respectively:

$$C' = \frac{s}{1-P} \tag{2}$$

$$C = \frac{s}{P} \tag{3}$$

where $s$ is the standard deviation and $P$ is the probability of an event.

The uncertainty propagation through the "OR" gate is given by Eq. 4 and Eq. 5 that calculate the probability and coefficient of variation of outputs, $P_{or}$ and $C'_{or}$, respectively:

$$P_{or} = 1 - \prod_{i=1}^{n}(1 - P_i) \tag{4}$$

$$1 + C'^{2}_{or} = \prod_{i=1}^{n}(1 + C'^{2}_{i}) \tag{5}$$

where $P_i$ and $C'_i$ are the probabilities and coefficients of variation of inputs, respectively.

For an "AND" gate, the probability and coefficient of variation of outputs, $P_{and}$ and $C_{and}$, are given by Eq. 6 and Eq. 7, respectively:

$$P_{and} = \prod_{i=1}^{n} P_i \tag{6}$$

$$1 + C^{2}_{and} = \prod_{i=1}^{n}(1 + C^{2}_{i}) \tag{7}$$

where $P_i$ and $C_i$ are the probabilities and coefficients of variation of inputs, respectively.

The comparison of probability density functions (pdfs) obtained to top event using MCS and method of moments was carried out using ANOVA (Analysis of Variance), for an alpha level of 0.05.

## 3. Results and Discussion

Fig. 3 shows an example of a flowchart of RENO software used for implementing MCS to propagate uncertainties through the fault tree shown in Fig 2, considering the pdf of basic events as normal and lognormal distributions (distribution parameters are based on case studies from literature [3]). Fig. 4 presents a comparison of probability density functions using method of moments and MCS. Using ANOVA, it is observed that in case of normal pdfs, the two distributions are not significantly different (Fig. 4a). Then the method of moments, which does not require expensive software and excessive run times as MCS, can be used in case of normal distributions. However, when are used other pdfs (e.g., lognormal), the differences between the two methods can be significant and MCS should be used (Fig. 4b). Moreover, the method of moments assumes that the top event pdf is also normal, which is not always true. MCS, through random sampling of basic event pdfs, get the exact distribution of top event, which can be fitted to more adequate distribution (e.g., lognormal, Weibull, gamma, etc.).

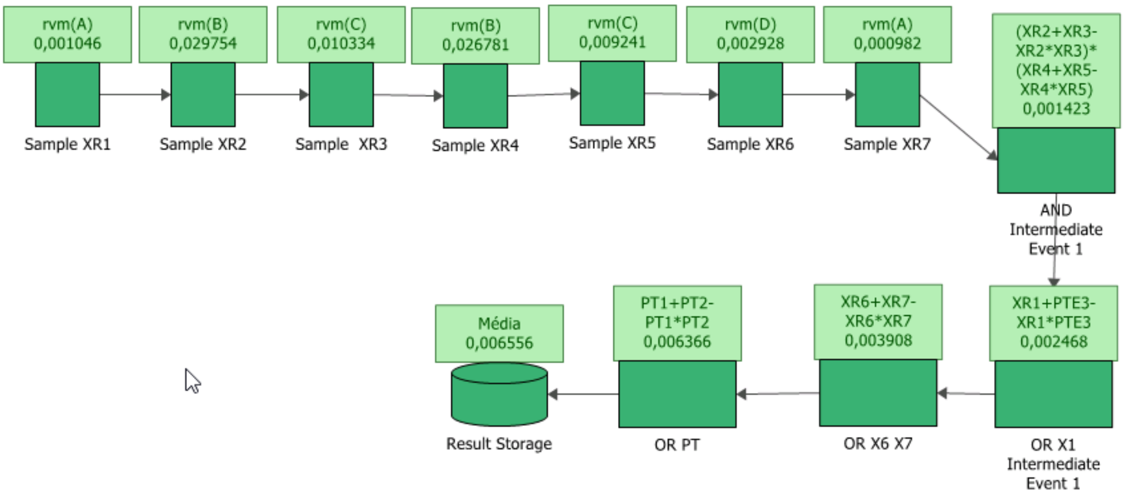


Figure 3: Example of a flowchart of RENO software used for implementing MCS for the fault tree shown in Fig 2.

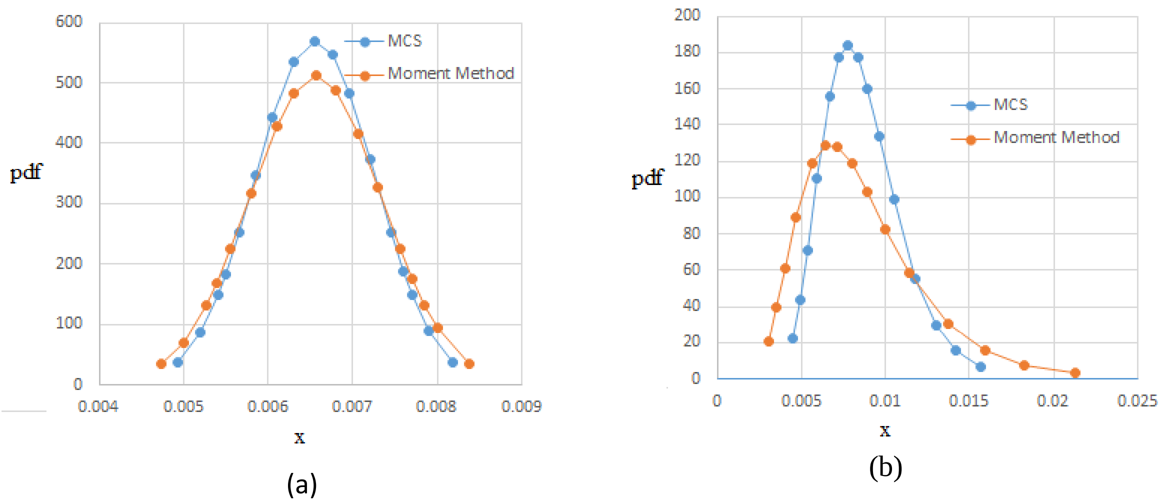


Figure 4: Comparison of probability density functions (pdfs) of top event of Fig. 2 using method of moments and MCS, for normal (a) and lognormal (b) distributions, respectively.

## 4. Conclusions

A comparison of method of moments and Monte Carlo simulation for uncertainty propagation in fault trees was presented. The advantages and disadvantages using different types of probability density functions were discussed and examples using normal and lognormal distributions were analyzed. This kind of uncertainty propagation in fault trees plays an important role when implementing PRAs in licensing processes of nuclear facilities, mainly in nuclear power plants, to evaluate if the plants meet risk acceptance criteria.


## Acknowledgements

The following Brazilian institutions support this work: Centro de Desenvolvimento da Tecnologia Nuclear (CDTN); Comissão Nacional de Energia Nuclear (CNEN); Financiadora de Inovação e Pesquisa (FINEP); Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq); and Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG).


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