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Extending the Nodal Expansion Method Basis Functions

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1. Introduction

In 1977 Finnemann, Bennewitz, and Wagner (Finnemann et al., 1977) introduced the Nodal Expansion Method (NEM), by combining neutron diffusion theory, polynomial expansions, interface currents continuity, and weighted residual techniques. Two classical two-dimensional benchmarks were simulated: IAEA and LRABWR. Considering assembly-size nodes, the fifth-order expansion using Galerkin weighting (G5) provided more accurate results than the fourth-order expansion using moments weighting (M4). Nevertheless, the authors suggested moments weighting to be preferable. The quadratic approximation has been considered appropriate to represent the transverse leakage terms.

Most nodal diffusion methods make use of a transverse integration procedure to solve the multi-dimensional neutron diffusion equation. The multi-dimensional problem is reduced to a set of coupled one-dimensional equations. NEM is a consistent method, converging to the exact solution as the mesh spacings are reduced, with no restrictions on energy groups, computationally efficient, and accurate for meshes up to the typical size of assemblies. NEM can be regarded as an application of the weighted residual techniques and its effectiveness depends on the proper choice of weight functions. In most LWR simulations, the flux has a relatively smooth shape. Nevertheless, as traditionally used by NEM, a fourth-order expansion may not produce sufficiently accurate results for nodes located near the core-reflector interface regions, mainly due to thermal neutron effects.

A detailed analysis of the Nodal Expansion Method has been performed, which allowed the development of formalism capable of generating polynomial expansions of any order. The proposed basis functions should introduce the higher-order expansions needed to describe the average transverse flux more properly. Comparisons for two- and three-dimensional LWR static benchmarks simulations have been performed using up to tenth-order polynomial expansions, for both weight functions: moments and Galerkin. The new basis functions have led to more accurate results for k_{eff} and normalized assembly power distribution, including nodes at fuel-reflector interfaces.

2. Methodology

In nodal diffusion methods, the reactor core spatial domain is partitioned into regular volumes, called nodes. In NEM, the node (m) is a rectangular parallelepiped with a coarse meshing comparable to the fuel assembly pitch. The integration of the *neutron continuity equation* over a node volume (V_m), in an average sense, provides a set of new equations called *nodal balance equations*.

Although the nodal balance equations formulation is performed without any approximation, it is incomplete in the sense that it relates two unknown quantities: the average node flux ($\bar{\phi}_g^m$), and interface net currents ($\bar{J}_{gus}^m$). To establish a complete system of equations, additional relations are derived from Fick's

Law by applying a transverse-integration procedure. The resulting equations are called *coupling equations*. The coupling equations express the relationship between the interface net currents and average transverse fluxes ($\bar{\psi}_{gu}^{m}(u)$). An equation describing the behavior of the average transverse flux along the u-direction is obtained from the transversely integrated neutron diffusion equation. The leakage term in the transversely integrated neutron diffusion equation is split into three components: one on the generic u-direction (considered direction) and two on the transverse directions ($L_{gu}^{m}(u)$).

In order to solve the transversely integrated neutron diffusion equation, NEM makes use of a fourth-order polynomial expansion to approximate $\bar{\psi}_{gu}^{m}(u)$ and a second-order expansion to $L_{gu}^{m}(u)$:

$$\bar{\psi}_{gu}^{m}(v) = \sum_{k=0}^{4} c_{kgu}^{m}\, h_k(v),\; L_{gu}^{m}(v) = \sum_{k=0}^{2} \alpha_{kgu}^{m}\, h_k(v), v = u/a_u^m \;\; (u \in [0, a_u^m], v \in [0,1]). \tag{1}$$

Where the $h_k(v)$ are NEM's basis functions (Table I), and a_u^m denotes de node width along u-direction.

Table I: NEM's basis functions.

k	$h_k(v)$
0	1
1	$2v-1$
2	$6v(1-v)-1$
3	$6v(1-v)(2v-1)$
4	$6v(1-v)(5v^2-5v+1)$
5	$6v(1-v)(2v-1)(6v^2-6v+1)$

Imposing consistent conditions to the average transverse flux polynomial expansion form, the first three expansion coefficients can be determined. The resulting equation is known as the *basic variant* of NEM. The final equations' set is obtained by inserting the average transverse flux approximation into the coupling equations and eliminating the outgoing currents from the nodal balance equation. Although the balance equations for nodal fluxes and interface currents have been established, the higher-order coefficients (c_{3gu}^{m} and c_{4gu}^{m}) remain unknown. For such a purpose, the weighted residual procedure can be applied.

Once all NEM steps were identified and understood, it became possible to generate the basis functions of any order and go through all the steps of the method. The new basis functions up to the tenth-order are shown in Table II.

Table II: New basis functions up to tenth-order.

k	$h_k(v)$
6	$6\,v(1-v)\left(\frac{308}{9}v^4-\frac{616}{9}v^3+\frac{419}{9}v^2-\frac{111}{9}v+1\right)$
7	$6v(1-v)(2v-1)\left(\frac{143}{3}v^4-\frac{286}{3}v^3+\frac{187}{3}v^2-\frac{44}{3}v+1\right)$
8	$6\,v(1-v)\left(\frac{5265}{18}v^6-\frac{15795}{18}v^5+\frac{18421}{18}v^4-\frac{10517}{18}v^3+\frac{3028}{18}v^2-\frac{402}{18}v+1\right)$
9	$6\,v(1-v)(2v-1)(442v^6-1326v^5+1521v^4-832v^3+221v^2-26v+1)$
10	$6\,v(1-v)\left(\frac{28424}{10}v^8-\frac{113696}{10}v^7+\frac{188479}{10}v^6-\frac{167501}{10}v^5+\frac{86179}{10}v^4-\frac{25835}{10}v^3+\frac{4300}{10}v^2-\frac{350}{10}v+1\right)$

3. Results and Discussion

In order to assess the potential of the proposed higher-order expansions, three computational simulations up to the tenth order were performed: IAEA 2D, BIBLIS 2D and LRABWR 3D. The benchmarks simulations were used to determine the effects of the proposed higher-order expansions on the calculation accuracy and effectiveness. In all simulated benchmarks, the size of the nodes was the same as the pitch of fuel assemblies (one node per assembly).

A FORTRAN 90 computer program, based on the nodal diffusion computational infrastructure (Silva et al., 2010) of the Nuclear Engineering Program (PEN) at COPPE/UFRJ, was developed using the Microsoft Visual Studio 2008 compiler and running on an Intel i3-2350M 64 bit 2.30GHz processor using the Windows 7 operating system. All floating-point calculations were performed in double precision to maximize accuracy.

The adopted tolerances were 10^{-8} for k_{eff} and 10^{-6} for the average nodal fluxes ($\bar{\phi}_g^m$) in all simulations, and a parabolic approximation for the transverse leakages ($L_{gu}^m(u)$) was used in all calculations.

The reference values, whenever possible, were those found in the scientific literature obtained from fine mesh calculations. Considering the easy availability of the benchmarks data, only the reference sources are listed (Table III).

The simulations are identified by a capital letter and a number. The letter denotes the type of weight function (Moments or Galerkin), while the number specifies the adopted expansion order. The M4 approximation is used in NEM.

Table III: Benchmarks source references.

IAEA 2D - Data reference: ANL-7416 Supplement 2, ID.11-A2 (*Benchmark Problem Book*, 1977).
Results reference: Pessoa (Pessoa et al., 2016), Finite Difference Method, 0.50 cm mesh size.

BIBLIS 2D - Data reference: Müller (Müller and Weiss, 1991).
Results reference: Shober (Shober, 1978).

LRABWR 3D - Data reference: Kim (Kim, 1983).
Results reference: Kim (Kim, 1983), Finite Difference Method, 0.75 cm mesh size.

Table IV: Summary of the simulated benchmarks results.

Benchmark	Type	Δt_{EXE} (sec)	Δk_{eff} (pcm)	ΔP_{Avg} (%)	ΔP_{Max} (%)	N_{WP} (%)	N_{PN} (%)
IAEA 2D	M4	0.28	-7.6	0.43	1.22	-	-
	M5	0.30	4.0	0.38	-1.58	56	36
	G6	0.31	0.7	0.20	-0.91	63	36
BIBLIS 2D	M4	0.33	7.8	0.42	2.32	-	-
	M5	0.37	15.6	0.23	1.18	70	100
	G6	0.35	17.7	0.28	1.25	61	83
LRABWR 3D	M4	6.18	-16.1	0.69	2.15	-	-
	M6	7.94	0	0.11	0.39	99	100
	G8	11.57	0	0.11	0.39	99	100

Δt_{EXE} : Execution time.
Δk_{eff} : Eigenvalue relative error. (pcm stands for "percent mille", 10^{-5}.)
ΔP_{Avg} : Average relative error for the normalized assembly power distribution.
ΔP_{Max}: Maximum relative error for the normalized assembly power distribution.
N_{WP} : Percent result favoring a higher-order expansion in the whole normalized assembly power distribution.
N_{PN} : Percent result favoring a higher-order expansion in peripheral nodes (fuel-reflector interface).

The fourth- and fifth-order expansions are the highest orders verified in the reference paper (Finnemann et al., 1977), using moments and Galerkin weightings respectively (M_2B^2 and G_3B^2, according to the Finnemann's notation, where B^2 refers to the parabolic approximation for transverse leakages). Consequently, M_2 and G_3 have been adopted as starting points in the present investigation (M4 and G5 in current notation, respectively).

In order to verify which higher-order expansion was in good (or the best) agreement to the expected normalized assembly power distribution, the average power deviation ΔP_{Avg} was taken as the most efficient indicator.

4. Conclusions

As known, in its traditional formulation, NEM does not work properly in regions bordering the reflector. The introduced basis functions have led to more accurate results for both k_{eff} and normalized assembly power distribution, including nodes at fuel-reflector interfaces. Although it looks promising, further studies on the improvements introduced by the extension of the basis functions should be done.

Considering the presented information, it is reasonable to claim that the higher-orders expansions enhance NEM's accuracy with no loss of efficiency. On the other hand, it should also be considered that this work is the initial investigation on the applicability and feasibility of the introduced basis functions. Therefore, further tests are still needed.

References

[1] Finnemann, H., Bennewitz, F., Wagner, M.R., "Interface current techniques for multidimensional reactor calculations," *Atomkernenergie*, ATKE 30, pp. 123–128 (1977).

[2] Silva, F.C., Alvim, A.C.M., Martinez, A.S., "An Alternative Nodal Expansion Method without Inner Iteration," *International Conference on the Physics of Reactors 2010 (PHYSOR 2010): Advances in Reactor Physics to Power the Nuclear Renaissance*, Pittsburgh, Pennsylvania, USA, May 9-14 (2010).

[3] Argonne Code Center, *Benchmark Problem Book. ANL-7416-Supplement 2, Mathematics and Computers* (UC-32), Computational Benchmark Problems Committee of the Mathematics and Computation Division of the American Nuclear Society, Argonne National Laboratory, USA (1977).

[4] Pessoa, P.I.O., Araujo, L.M., Silva, F.C., Menezes, A.A.M., "Numerical Methods Applied to Pin Power Reconstruction Based on Coarse-Mesh Nodal Calculation," *Ann. Nucl. Energy*, vol. 118, pp. 291-312 (2018).

[5] Müller, E.Z., Weiss, Z.J., "Benchmarking with the Multigroup Diffusion High-Order Response Matrix Method," *Ann. Nucl. Energy*, vol. 18 (9), pp. 535–544 (1991).

[6] Shober, R.A., *A Nodal Method for Solving Transient Fewgroup Neutron Diffusion Equations* (ANL-78-51), U.S. Department of Energy, Argonne National Laboratory, Applied Physics Division (1978).

[7] Kim, C.H., "A Modified Borrensen's Coarse-Mesh Solution to the LRA-BWR Benchmark Problem," **Journal of the Korean Nuclear Society**, vol. 15 (2), pp. 135–141 (1983).