GeoAmericas 2020
4th PAN AMERICAN CONFERENCE ON GEOSYNTHETICS
26-29 APRIL 2020 • RIO DE JANEIRO • BRAZIL

# Simulation of prefabricated thermal drains in soft clay

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ABSTRACT

This paper focuses on the thermo-hydro-mechanical behavior of soft clay surrounding a prefabricated thermal drain. A prefabricated thermal drain combines features of a conventional prefabricated vertical drain (PVD) and a closed-loop geothermal heat exchanger by placing plastic tubing within the core of the PVD through which heated fluid can be circulated. The prefabricated thermal drain can be used to increase the temperature of the surrounding soft clay, which will generate excess pore water pressures due to differential thermal expansion of the pore fluid and clay particles. As these excess pore water pressures drain, the soft clay will experience volumetric contraction. The elevated temperature leads to an increase in the hydraulic conductivity and the volumetric contraction leads to an increase in thermal conductivity, making this a highly coupled process. Although thermal drains have been tested in proof of concept field experiments, there are still several variables that need to be better understood. This paper presents numerical simulations of the coupled heat transfer, water flow, and volume change in the soft soil surrounding a prefabricated thermal drain that were validated using the results from large-scale laboratory experiments. Numerical simulations were found to agree well with the experimental data. A further analysis on the performance of the thermal PVD indicates an increase in surface settlement with an increase in drain temperature and a significant reduction in the surcharge required when using a thermal PVD.

## 1. INTRODUCTION

Construction on soft clays poses a challenge to geotechnical engineers due to issues including undesirable post-construction settlements. Prefabricated vertical drains (PVD) are widely used in conjunction with a surcharge as a method of improving soft clays prior to construction. A PVD is a type of geosynthetic consisting of a perforated core material wrapped with a nonwoven geotextile, and is typically used to expedite consolidation of slow draining soils. Although the use of PVDs helps accelerate the consolidation process by shortening the drainage path for dissipation of excess pore water pressures generated by application of a surcharge, the time required to complete primary consolidation can still be significant in some instances. Furthermore, maintaining stable surcharge embankments on soft clays and the cost of installation of PVDs are other challenges associated with this ground improvement technique. To overcome these obstacles, methods involving the use of vacuum pressure and heat combined with a surcharge have been studied by several researchers (Kjellman 1952, Abuel-Naga et al. 2006) where an improvement in the rate and magnitude of consolidation have been observed. Combining geothermal heat exchangers with PVDs may be a way to achieve this purpose and also offset the high installation costs by using the PVDs for long-term geothermal heat exchange after ground improvement is complete.

This paper presents a simulation of the thermo-hydro-mechanical behavior of a saturated normally consolidated soft clay surrounding a prefabricated thermal drain considering the effects of the applied temperature change and surcharge on the consolidation process. A theoretical formulation which accounts for the coupled thermal, hydraulic and mechanical behavior of normally consolidated clay is presented and a numerical model is developed to simulate these processes. The simulated results are compared with experimental data from literature and a further analysis on the efficiency of thermal vertical drains with respect to the magnitude of temperature and surcharge applied are presented.

## 2. BACKGROUND

A thermal prefabricated vertical drain (i.e., a thermal PVD) combines features of a conventional PVD and a geothermal heat exchanger by placing closed-loop plastic tubing within the core of the PVD through which heated fluid can be circulated. Experimental investigations have shown that an increase in temperature will also increase the settlement observed in soil. Bergenstahl et al. (1994) measured the surface settlements and pore pressures at a test field for ground heat storage in Linkoping, Sweden. Increases in settlement and excess pore water pressure were observed as a result of increasing the ground temperatures up to 70 °C. Abuel-Naga et al. (2006) proposed the use of thermal PVDs as a method of ground improvement. An increase in the magnitude as well as the rate of consolidation was observed when a thermal PVD was used in combination with a surcharge. Similar observations were also made by Pothiraksanon et al. (2010), Artidteang et al. (2011) and Salager et al. (2012) in field and laboratory tests.

The increase in the rate of consolidation is attributed to thermally induced excess pore water pressure and the increase in hydraulic conductivity of the pore fluid. Many researchers have observed excess pore water pressure generation at elevated temperatures under undrained or partially drained conditions as a result of the differential expansion of the soil particles and the pore fluid (Campanella and Mitchell 1968, Hueckel and Pellegrini 1992, Abuel-Naga et al. 2006). Campanella and Mitchell (1968) expressed the effect of temperature (T) on excess pore water pressure (Δu) as follows.

$$\Delta u = \frac{n\Delta T(\alpha_s - \alpha_w) + \alpha_{st}\Delta T}{m_v} \quad [1]$$

where n is the porosity, $\alpha_s$ is the volumetric thermal expansion coefficient of soil grains, $\alpha_w$ is the volumetric thermal expansion coefficient of water, $\alpha_{st}$ is the physico-chemical coefficient of structural volume change due to temperature and $m_v$ is the compressibility of the soil structure. As these excess pore water pressures dissipate, the soil will undergo volumetric contraction.

Furthermore, an increase in temperature will also contribute to an increase in the hydraulic conductivity of the pore fluid due to a decrease in viscosity. The hydraulic conductivity (k) can be related to the fluid and soil properties as shown in Equation 2.

$$K = \frac{k\,\eta_w}{\rho_w g} \quad [2]$$

where K is the intrinsic permeability, $\eta_w$ is the dynamic viscosity of the fluid, $\rho_w$ is the fluid density and g is the coefficient of gravity. Abuel-Naga et al. (2006) observed an increase in hydraulic conductivity with an increase in temperature from constant head flexible wall hydraulic conductivity tests conducted on soft Bangkok clay. However, the intrinsic permeability was found to be independent of temperature.

The effect of temperature on volume change of soft clays has been broadly investigated by several researchers (Campanella and Mitchel 1968, Baldi et al. 1988, Hueckel and Baldi 1990, Sultan et al. 2002, Laloui and Cekerevac 2003, Abuel-Naga et al. 2006). Plastic contractive volumetric changes are observed in normally consolidated clays when subjected to drained heating, while elastic expansive volumetric changes are observed in over-consolidated clays during drained heating, albeit with a possibility for plastic contraction at higher temperatures (Cui et al. 2000). Therefore, it is evident that drained heating could also have an impact on the magnitude of settlement in a ground improvement application. The use of thermal PVDs in normally consolidated clay deposits can be expected to produce higher overall settlements than those obtained using conventional PVDs. Several constitutive models exist in the literature to reflect the thermo-mechanical behavior of soft soils (Hueckel and Borsetto 1990, Cui et al. 2000, Laloui and Cekerevac, 2003, Abuel-Naga et al. 2006). These constitutive models attribute the generation of thermal strains to a thermal yielding mechanism where a thermal yield limit is defined for the soil. Thermo-elastic strains are obtained at stress levels below the thermal yield limit and thermo-plastic strains are obtained at stress levels above the thermal yield limit. For isothermal conditions, the yield limit will be equal to the current mean effective stress for normally consolidated soils. Drained heating will result in a reduction in the yield limit causing the virgin compression line (VCL) to shift to the left and this shift will correspond to a plastic contraction.

## 3. THEORETICAL FORMULATION

### 3.1 Soil domain geometry and boundary conditions

A schematic diagram of the thermal PVD arrangement in a finite soil domain is shown in Figure 1. The thermal PVD is inserted at the center of a cylindrical specimen of height h and radius r. A surcharge is applied at the top of the specimen. When a saturated soil layer is subjected to a surcharge, excess pore water pressures will be generated in the soil. As a result of the hydraulic gradient created, water will continue to flow radially towards the vertical drain. When the excess pore water pressure dissipates, volumetric contraction will occur in the soil.

In the thermal PVD application, an elevated temperature will be applied at the drain location and the domain boundary will be at room temperature. As the temperature of the soil increases with heat flow, the fluid flow will be impacted by the thermally induced excess pore water pressures and the increased hydraulic conductivity. Furthermore, thermal plastic strains will be generated as discussed in the previous section. For the simulations presented in this study, heat transfer and fluid flow were considered to be axisymmetric about the axis of the drain. Assuming the variation of temperature to be uniform in the vertical direction the geometry was simplified to a soil layer along a radius. A saturated normally consolidated clay was modeled considering the coupled processes of heat transfer, water flow and volume change.

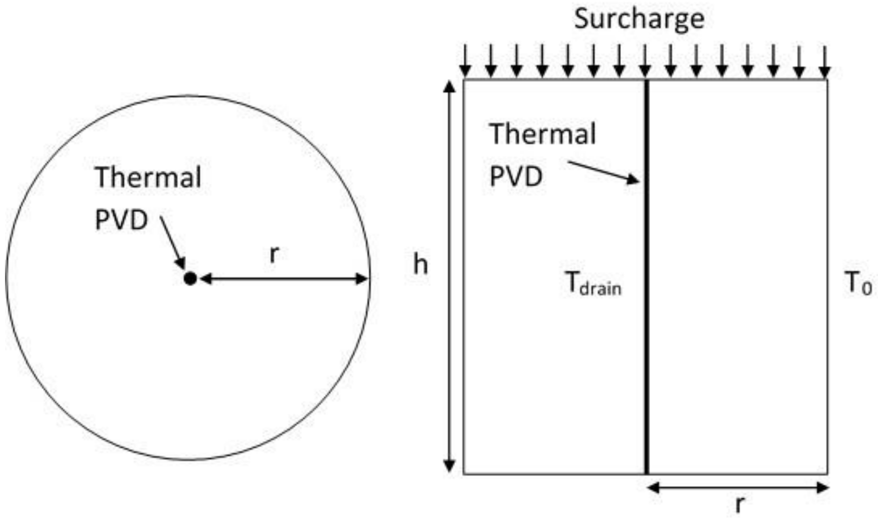


Figure 1. Schematic diagram of the thermal PVD arrangement in a finite soil domain.

### 3.2 Heat transfer

The temperature of the surrounding clay will be increased when a thermal PVD is used. To account for the impact of temperature on fluid flow and volume change, spatial and temporal variation of temperature will have to be obtained. The thermal PVD was considered as a line heat source and the vertical distribution of temperature was assumed to be uniform. It was also assumed that heat transfer through the soil medium will occur through conduction only. Therefore, the governing equation for conductive radial heat transfer through soil based on Fourier's law and conservation of energy will be simplified as follows in cylindrical coordinates.

$$\frac{\rho_s C}{\lambda_T}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \quad [3]$$

where $\rho_s$ is the total density of soil, C is the specific heat capacity of the soil, $\lambda_T$ is the thermal conductivity of the soil, T is the temperature and r is the radial distance.

Volumetric contraction of the clay will lead to an increase in thermal conductivity. The change in thermal conductivity with porosity was considered using a parallel model as follows (Dong et al. 2015):

$$\lambda_T = n\lambda_f + (1-n)\lambda_s \quad [4]$$

where $\lambda_f$ is the thermal conductivity of the pore water and $\lambda_s$ is the thermal conductivity of the soil particles.

### 3.3 Fluid flow

Fluid flow through the soil mass can be modeled based on the principle of mass conservation. The governing equation in cylindrical coordinates will be reduced as shown for the problem geometry considered.

$$\frac{\partial(n\rho_w)}{\partial t} = -\frac{1}{r}\frac{\partial(r\rho_w v)}{\partial r} \quad [5]$$

where v is the fluid velocity. Fluid velocity for a porous medium can be expressed using Darcy's law as follows:

$$v = -\frac{K}{\eta_w}\frac{\partial U}{\partial r} \quad [6]$$

where U is the pore water pressure. By substituting Equation 6 into Equation 5 and using the product rule, the following equation can be obtained.

$$n\frac{\partial \rho_w}{\partial t} + \rho_w\frac{\partial n}{\partial t} = \frac{K}{\eta_w}\left(\rho_w\frac{\partial^2 U}{\partial r^2} + \frac{\rho_w}{r}\frac{\partial U}{\partial r} + \frac{\partial U}{\partial r}\frac{\partial \rho_w}{\partial r}\right) \quad [7]$$

The fluid density is not a constant, but will vary as a function of temperature, as shown in Equation 8 and the fluid viscosity can be expressed as a function of temperature as in Equation 9 after Hillel (1980).

$$\frac{\partial \rho_w}{\partial t} = -\rho_w \alpha_w \frac{\partial T}{\partial t} \quad [8]$$

$$\eta_w(T) = -0.00046575 \ln(T) + 0.00239138 \quad [9]$$

By substituting Equation 8 into Equation 7 and considering the spatial variation of fluid density to be negligible, a general equation for non-isothermal fluid flow through porous media is given as follows:

$$-n\alpha_w \frac{\partial T}{\partial t} + \frac{\partial n}{\partial t} = \frac{K}{\eta_w}\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) \quad [10]$$

## 3.4 Volume change

For a normally consolidated soil, the volume change due to mechanical loading can be obtained using the compressibility relationships in terms of void ratio, as follows:

$$\partial e_m = \lambda \frac{\partial \sigma'}{\sigma'} \quad [11]$$

where e is the void ratio of the soil, λ is the slope of the VCL and σ´ is the mean effective stress. As described in the previous section, normally consolidated soils will also be subjected to thermo-plastic volumetric changes during heating. The following relationship proposed by Laloui and Cekerevac (2003) was used to obtain the thermal volumetric changes in terms of void ratio, as follows:

$$\partial e_T = \frac{(1+e_0)\gamma\, \partial T}{2.303\beta T(1-\gamma \log\left(\frac{T}{T_0}\right))} \quad [12]$$

where $\gamma$ is a material parameter, $e_0$ is the initial void ratio, β is the inverse of plastic compressibility and $T_0$ is the room temperature.

# 4. NUMERICAL MODEL

To simulate the coupled phenomena described in the previous section, a numerical model was developed using the finite difference method. A vertical thermal drain embedded in a saturated normally consolidated soil layer was represented using an axisymmetric radial flow simulation. The soil layer along a radius was spatially discretized into elements of equal size. The thermal drain is treated as a line heat source where a constant temperature boundary is applied at the drain location. The far field boundary can be represented based on the experimental setup. For example, a constant temperature boundary condition or a function accounting for daily temperature fluctuations can be used when the far field is maintained at ambient temperature. On the other hand, a convective boundary condition can be used to account for heat loss or zero heat flux to model thermal insulation at the boundary. A similar approach can also be used to model the surface temperatures. As mentioned in the previous section, the vertical distribution of temperature was assumed to be uniform and the analysis was simplified to a one dimensional radial heat transfer problem. The water pressure at the drain was also maintained constant at hydrostatic pressure and the far field velocity was considered to be zero. The initial temperature was taken as equal to ambient temperature and the initial porosity was determined based on the soil. The initial pore water pressure at the time of heating was determined based on hydrostatic conditions and the excess pore water pressure associated with the applied surcharge.

Based on the initial and boundary conditions, temperature, pore water pressure and settlement was calculated for each time step. A central difference scheme was used in the spatial domain whereas a forward difference scheme was used in the time domain. Both steady-state and transient evaluations of the soil behavior can be conducted using this simple geometry and it is possible to obtain the variables of interest as a function of both space and time using the numerical model. The domain geometry and the numerical formulation was implemented and solved using Matlab.

# 5. COMPARISON WITH EXPERIMENTAL DATA

The simulated thermo-hydro-mechanical behavior of a saturated normally consolidated clay layer was compared with experimental data from literature. Artidteang et al. (2011) investigated the performance of a single thermal PVD in soft Bangkok clay using a large-scale consolidometer having similar boundary conditions to the schematic in Figure 1. The diameter of the consolidometer was 0.45 m and soil was filled up to a height of 0.7 m. The specimens were first allowed to reach 90% consolidation under a surcharge of 50 kPa. For the specimen using a conventional PVD, an additional surcharge of 50 kPa was applied whereas for the specimen with a thermal PVD, a 50 kPa surcharge and heat up to 90 °C was applied simultaneously. To simulate this setup, an axisymmetric domain with a length of 0.225 m was considered and divided into elements of size 0.0225 m. A constant temperature of 90 °C was applied at the boundary with the thermal PVD and a convective boundary was imposed at the edge of the consolidometer. The initial temperature was taken as 25 °C and the initial porosity was considered to be 0.65. The hydrostatic pressures were determined considering a mid-depth of the soil specimen. The material parameters for Bangkok clay used in the numerical model were based on data from Artidteang et al. (2011) and Abuel-Naga et al. (2006) and are summarized in Table 1.

Table 1. Material parameters for Bangkok clay (Artidteang et al. 2011, Abuel-Naga et al. 2006).

| Parameter | Value |
|---|---|
| Total unit weight (kN/m$^3$) | 14.7 |
| Initial porosity | 0.65 |
| λ (slope of VCL) | 0.59 |
| κ (slope of RCL) | 0.1 |
| γ (soil parameter) | 0.43 |
| Thermal conductivity of soil particles (W/m/°C) | 1.9 |
| Thermal conductivity of pore water (W/m/°C) | 0.6 |
| Specific heat capacity (J/kg/°C) | 1500 |
| Intrinsic permeability (m$^2$) | 0.6x10$^{-16}$ |

The predicted time series of temperature are shown in Figure 2 along with the corresponding data from Artidteang et al. (2011). Close agreement can be observed between the experimental results and the numerical simulation, specifically for locations further away from the thermal drain. A slight underestimation of temperature is seen at locations closer to the drains. The differences could be due to convective heat transfer not being considered and the assumed values of certain material properties. A comparison of simulated results for the settlement of the clay specimen with experimental data is presented in Figure 3 where the use of a conventional PVD and a thermal PVD is considered. The simulated results closely match the experimental data where the increase in both the rate and magnitude of settlement is captured where a thermal PVD was being used. Figure 4 shows the experimental data and the predicted results for the excess pore water pressure generated when using a thermal PVD. Although a difference in the magnitude of the maximum excess pore water pressure is observed, the numerical model seem to simulate the trend in dissipation of excess pore water pressure well.

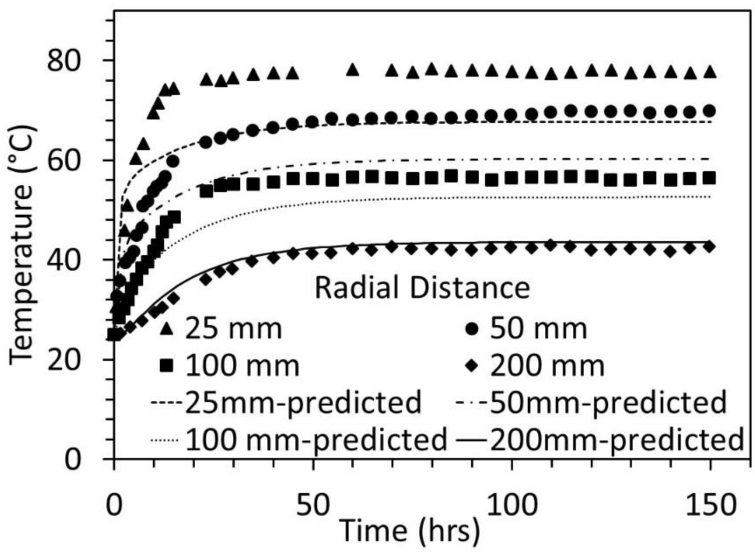


Figure 2. Comparison of time series of temperature with data from Artidteang et al. (2011).

The numerical formulation was also used to simulate the experimental results of Salager et al. (2012) for Sion silt specimens in a large consolidometer having a diameter 0.3 m and height 0.6 m, also having similar boundary conditions to those shown in Figure 1. This setup was simulated using an axisymmetric domain of length 0.15 m divided into elements of size 0.015 m. The material parameters for Sion silt are given in Table 2. Two soil specimens were tested, one at 25 °C and the other at 43 °C and a surcharge of 50 kPa was applied. For the specimen at 43 °C, the surcharge was applied after the specimen reached a steady-state at the elevated temperature in contrast to simultaneous application of heat and surcharge. Under these conditions, the impact of temperature on the rate of consolidation was more pronounced than on the magnitude of settlement, where the latter was observed to be similar to that at ambient temperature. The predicted values for settlement along with the corresponding experimental data are shown in Figure 5. The results at 25 °C are slightly over-predicted by the model whereas the values at 43 °C shows good agreement. The numerical simulation capture the thermo-hydro-mechanical processes surrounding a thermal PVD well, indicating that it will be useful in a preliminary analysis of the different variables that affect the performance of thermal PVDs.

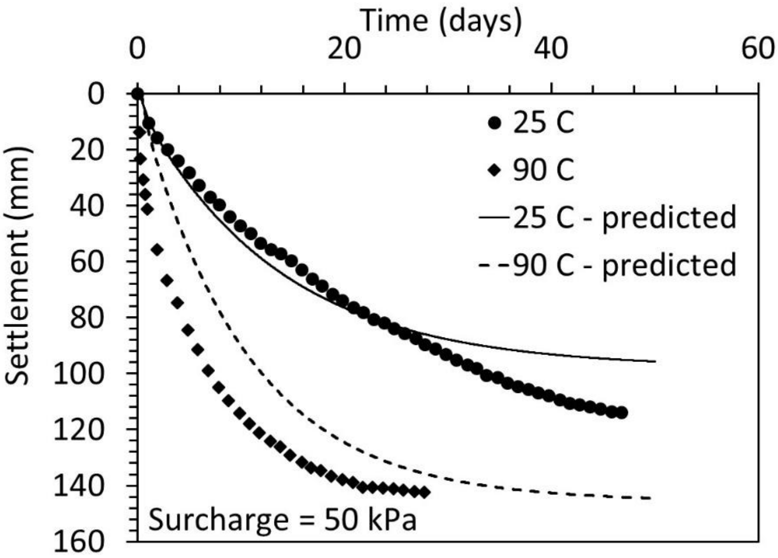


Figure 3. Comparison of simulated results for settlement with data from Artidteang et al. (2011).

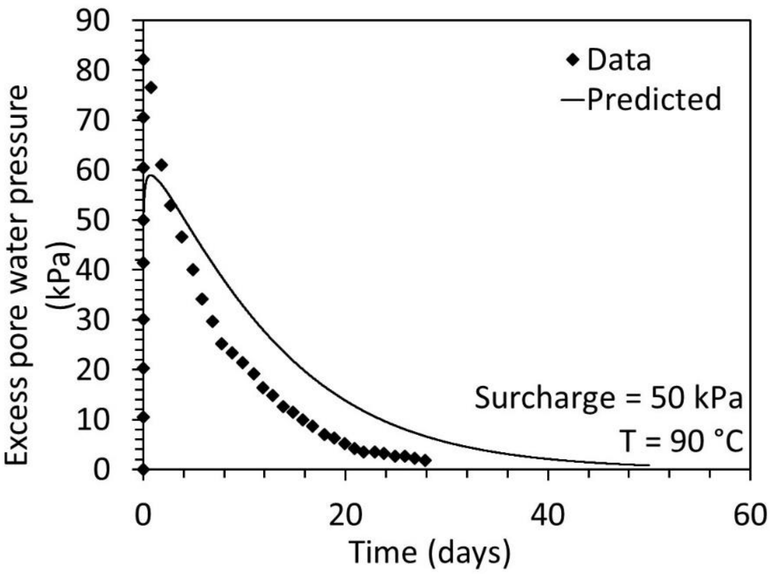


Figure 4. Comparison of excess pore water pressure generated with data from Artidteang et al. (2011).

Table 2. Material parameters for Sion silt (Salager et al. 2012, Laloui et al. 1997, Geiser et al. 2006).

| Parameter | Value |
|---|---|
| Total unit weight (kN/m$^3$) | 24.35 |
| Initial porosity | 0.44 |
| λ (slope of VCL) | 0.025 |
| κ (slope of RCL) | 0.007 |
| γ (soil parameter) | 0.46 |
| Thermal conductivity of soil particles (W/m/°C) | 1.5 |
| Thermal conductivity of pore water (W/m/°C) | 0.6 |
| Specific heat capacity (J/kg/°C) | 1500 |
| Intrinsic permeability (m$^2$) | 1x10$^{-14}$ |

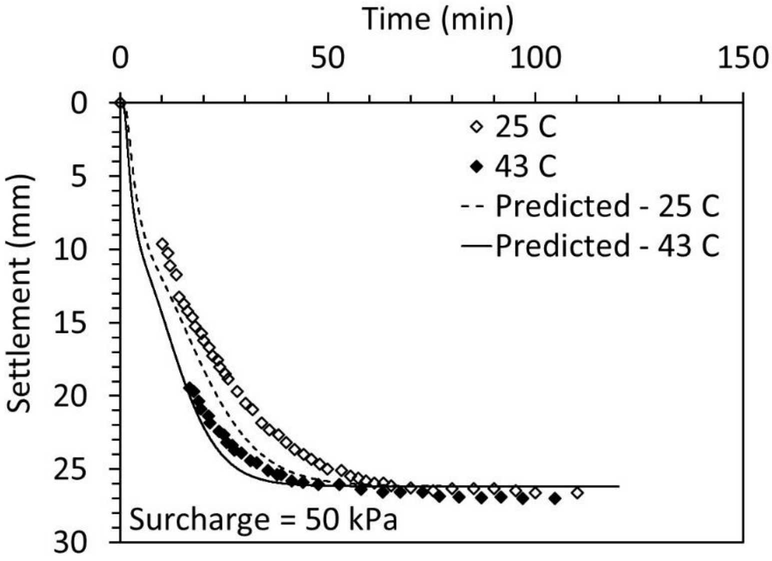


Figure 5. Comparison of simulated results for settlement with data from Salager et al. (2012).

## 6. ANALYSIS AND DISCUSSION

To further investigate the performance of thermal PVDs, the numerical model was used to simulate the soil behavior by considering different variables that affect the performance of the thermal PVD. Specifically, the effect of the magnitude of applied temperature and surcharge loads on consolidation settlement was investigated using the soil properties for Bangkok clay. A soil specimen with a height of 0.7 m and a diameter of 0.45 m was considered. The results obtained for consolidation settlement under a surcharge of 50 kPa are shown in Figure 6 for a thermal PVD having different magnitudes of temperature. As expected, an increase in the magnitude as well as the rate of settlement is observed as the temperature of the thermal PVD is increased.

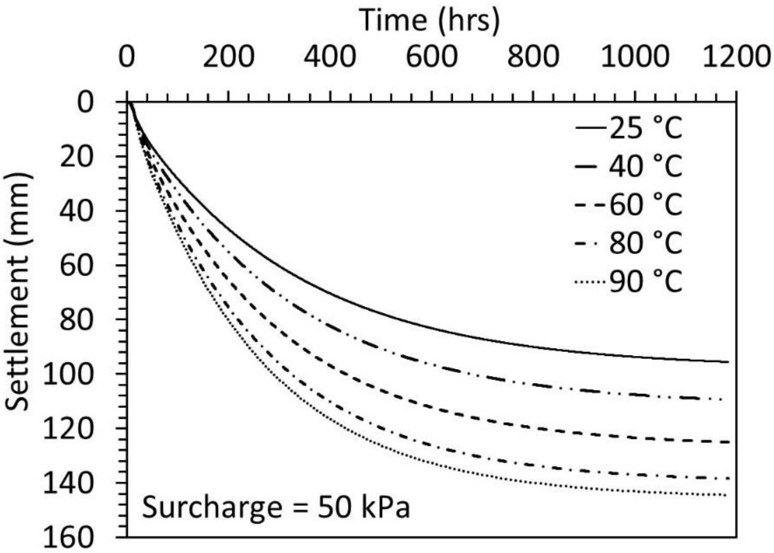


Figure 6. Effect of temperature on settlement for a thermal PVD combined with 50 kPa surcharge.

The settlements obtained for a soil layer with a single thermal PVD under different surcharge loads were also compared with those obtained using a conventional PVD. A constant temperature of 90 °C was applied at the thermal drain in this simulation. A comparison of the settlements obtained using a conventional PVD (25 °C) and a thermal PVD (90 °C) is shown in Figure 7 for surcharge levels of 10, 25, 50 and 100 kPa. As seen from the results in Figure 7, an increase in the magnitude of settlement is evident when a thermal PVD is used in place of a conventional PVD. Based on the simulation, by increasing the temperature up to 90 °C, the required surcharge can be reduced to almost half of that required when using a conventional PVD. The results obtained for the maximum settlement for each surcharge value considered is summarized in Figure 8. The amount of settlement resulting from thermo-plastic contractions is almost the same at each surcharge value as set out by the thermo-mechanical constitutive models. However, the contribution of thermal volume change on the total settlement increases as the applied surcharge is reduced. The increase in rate of consolidation can also be observed from the simulated results, as shown in Figure 9 for a surcharge of 50 kPa. The magnitude of final settlement achieved at the end of consolidation with a conventional PVD, is achieved about 4 times faster when using a thermal PVD for this specific simulation.

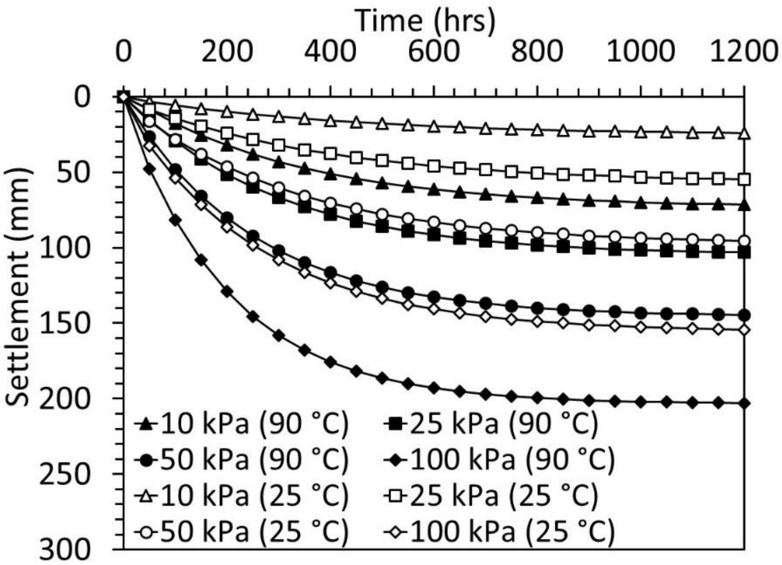


Figure 7. Comparison of settlement curves obtained at different surcharge levels.

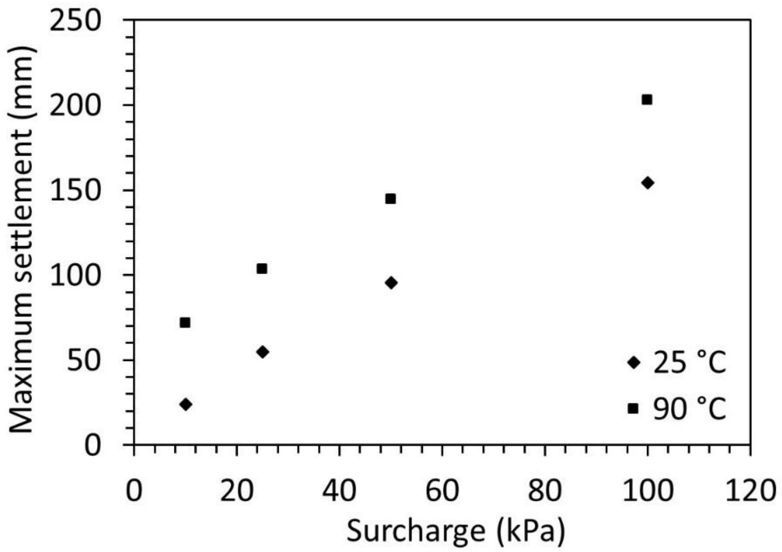


Figure 8. Effect of temperature and surcharge on the final magnitude of settlement.

Based on the analysis in Figures 8 and 9, the use of a thermal PVD has the potential to reduce the surcharge load requirement by approximately 50% and the time for consolidation is significantly reduced. The analysis in this paper was limited to the thermal behavior surrounding a single PVD. However, in field applications the use of multiple PVDs may have to be considered. Although not within the scope of this paper, this analysis can be extended to a case with multiple thermal PVDs by considering the intersection of thermal and hydraulic influence zones surrounding each PVD. In addition, while not considered in this paper, the future cost savings associated with using the PVDs after ground improvement to provide heat exchange or heat storage may help further justify the use of this emerging technology.

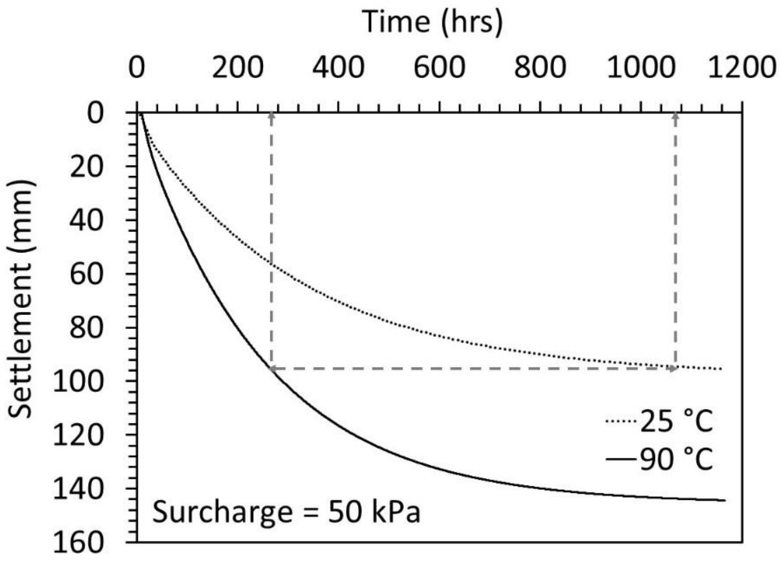


Figure 9. Effect of temperature on the rate of settlement.

## 7. CONCLUSION

The results from an investigation into thermo-hydro-mechanical behavior of a soft clay surrounding a thermal prefabricated vertical drain (PVD) were presented in this study. A theoretical framework was formulated representing the coupled interactions of heat and fluid flow and volume change in a saturated normally consolidated clay. The soil behavior adjacent to a thermal PVD was simulated using a numerical model. The numerical model simulations were found to agree well with the limited amount of experimental data available in literature on this topic. As observed in both experimental data and simulated results, the use of a thermal PVD has a significant effect on the consolidation process of a soft clay. A parametric analysis conducted on the performance of a thermal PVD shows that the required amount of surcharge can be reduced significantly when using a thermal PVD in lieu of a conventional PVD. A higher magnitude of temperature was observed to result in higher magnitudes of settlement and faster rates of consolidation.

The use of thermal PVDs was demonstrated to be a promising method for ground improvement as it could significantly reduce the time cost for consolidation as well as the required amount of surcharge. It was observed that a thermal PVD and a surcharge used in combination will yield a maximum output with respect to both magnitude and rate of settlement. An optimum combination of applied temperature change and surcharge can be determined based on factors such as soil geometry, soil properties and initial conditions, structural load, financial and time constraints. Further analysis can be carried out considering different soil types and the use of multiple drains where the optimal drain arrangement and depth can be determined.

## ACKNOWLEDGEMENTS

Funding from NSF grant CMMI 1941571 is appreciated. The opinions are those of the authors.

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