Accuracy of the Theory of Equivalent Thickness in Two-Layer System Applications with Geocell-Reinforced Soils

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ABSTRACT
Geocells are geosynthetics mainly applied to shallows foundations and pavement reinforcements in geotechnical engineering. Its structure confines the infill soil, usually granular, generating a layer with improved behavior in terms of resistance (ultimate state) and displacements (service state). A possible methodology to model this geocell-reinforced layer is considering it as a composite with equivalent geomechanics parameters over the foundation or subgrade - which is usually a soil with reduced resistance/stiffness. This kind of situation configures a two-layer system, mathematically treated in a rigorous form by the theory of elasticity (TE) and by the theory of equivalent thickness (TET), an approximated method. This article makes a comparison among these theories and a multilayer analysis software evaluates the applicability and accuracy of these methods, especially considering the usual variations in the problem as regards geometry and elastic parameters. The analysis shows that the approximated solutions by TET yield compatible results as compared with the rigorous formulation and with the software, proving the viability of its application mainly for being simple and easy to employ and propose and adjust of the approximated method in its coefficients to enhance the fit with the TE.

1. INTRODUCTION

1.1 Geocell reinforced structures studied by Plate Load Tests (PLTs)

A way to improve soil resistance and deformability is by using geocells (GCE), tridimensional geosynthetics made from the combination of strip plan elements joined by welding or sewing, commonly honeycomb-like, improving the confinement. This reinforced soil layer parameters are enhanced by an increment of confinement stress (Rajagopal, 1999; Madhavi Latha, 2000). Furthermore, there is the insertion of tension-resistant polymeric material, forming a composite material with a different behavior and characteristics as compared to infill soil.

Several researchers have made static or dynamic PLTs, with a sort of plate formats (circular, rectangular, strip) and soil type of the subgrade (sand or clay) to study the bearing capacity of the soil with this kind of reinforcement. Many variables have been changed in these studies: the infill soil (Hedge and Sitharam, 2015a; Mehjardi et al., 2019), relative density of the same infill granular soil (Shadmand et al. 2018), undrained resistance of the subgrade (Biswas et al., 2013, Biswas and Krishna 2016), geocell material (Hedge and Sitharam, 2015b), quantities of geocell layers (Khalaj et al., 2015), in addition to the change in dimensional characteristics of the geocell layers and of the footing (Figure 1). The results of these studies point out that the displacement is reduced for the same level of loading and the ultimate bearing capacity is increased when the geocell reinforcement is installed over a weak/smoother soil or the same sort of the infill soil. This enhanced stiffness is expressed by the bearing capacity improvement factor (I_f) that is the ratio of the loading pressure on a reinforced soil over a load of the unreinforced situation for the same level of displacement (Equation 1):

$$I_f(s/B) = p_{reinforced}(s/B)/p_{unreiforced}(s/B) \quad [1]$$

To model the structure of reinforced soil, the TE could be adopted to calculate small deformations. For this, it must be modeled as a semi-infinite mean with two horizontal layers. The inferior layer represents the subgrade and the superior one represents the reinforced soil.

Burmister (1943) derived a mathematical formulation for this situation, adopting a circular plate load and produced an abacus to calculate the maximum deflection of a structure, considering a range of the ratio of the Young modulus of the layer material between 2 and 10000, and both of them with the Poisson coefficient equal to 0.5. Other authors developed similar abacus and tables considering two layers (de Barros, 1966; Burmister, 1962) and three layers (Peattie, 1962; Jones, 1962; Burmister, 1966, Ueshita and Meyerhof, 1967) to determine deflections and stress for some combinations of v_1 and v_2 (and v_3 when three layers).

With the advances in computation, the theory developed by Burmister and others was implemented in software (for example, see Uzan, 1994; Hayhoe, 2002; Khazanovich and Wang, 2007), allowing the sizing of pavement structures with multiple layers by mechanistic analysis and the back-calculation of the material elastic parameters (Anderson, 1990).

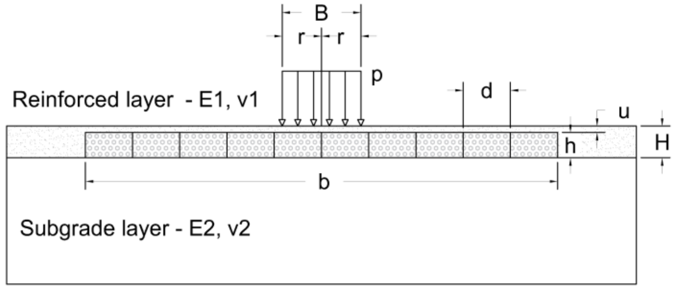


Figure 1 - Geometric and elastic parameters in a circular plate load test on a geocell reinforced soil

1.2 Theory of Equivalent Thickness (TET)

Palmer and Barber (1940) defined the concept of equivalent thickness: a layer with a height H and elastic modulus E_1 and Poisson coefficient v_1, laid over a semi-infinite layer with elastic parameters E_2 and v_2 has a correspondent height H_{eq} with the same stiffness, formed with the material of the inferior layer. Therefore, using the formulations derived by the TE, it is possible to estimate the stress state and the deflections. Odemark (1949) also developed this concept, and adopted the definition of the equivalent thickness shown in Equation 2, changing it and inserting a correction factor n:

$$H_{eq} = n \times H \times [(E_1/E_2) \times (1 - \upsilon_2^2)/(1 - \upsilon_1^2)]^{1/3} \quad [2]$$

By using the formulations derived by Boussinesq and Love, Odemark (1949) derived a method to estimate the deflection and radius curvature to two-layers system structures with different material whose coefficient of Poisson is equal to 0.5 for both the layers when it is loaded by a circular and flexible plate. The author stated that the estimation of the vertical normal stress on the axis of the load is possible, considering a conversion of the depth, with good agreement. For radial normal stress and maximal shear stress, the author mentioned that the application of Boussinesq's formula will yield discrepant values; however, for relations of $E_1/E_2 \leq 30$, the error is not excessive. For radial normal stress, the bottom fibers of the upper layer suffer tensions that are not possible to analyze by using this method directly.

Others authors have also applied the TET for mechanistic analysis and even for analyzing geocell reinforced soils structures, and modified to adapt to other loading situations, such as different plate formats, embedment of the footing and different combinations of Poisson coefficient (de Barros, 1966; Horak, 1988; Hirai, 2006; Dhar and Tarefder, 2011; Tafreshi et al., 2015; Avesani Neto, 2019).

It is currently agreed that the Poisson coefficients of the layers must be different, especially for geocell-reinforced ones. For subgrades consisting of saturated soft clay, for dynamic loads, as occurs on roads, it is possible to assume this coefficient is equal to 0.5 (undrained behavior, without volume variation), when the geocell reinforced layer filled with granular material and even a sandy or a stiff clayed subgrade must have its coefficient in a range between 0.2 and 0.4. Due to the confining effect of the geocell, the Poisson coefficient of the reinforced layer must be different of an unreinforced granular soil. Given that, Avesani Neto (2019) derived a generalization of the formulation proposed by Palmer and Barber (1940) and Odemark (1949) for the deflection of a two-layer system about their coefficients of Poisson, aiming to use it to perform back-analysis on PLT made on geocell-reinforced soils. Taking the definition of the equivalent thickness in Equation 2, the displacement at a center of a circular foot with radius r that applies a vertical stress p is given by Equations 3 to 6:

$$s = s_1 + s_2 = F' \times 2pr(1 - \nu_1^2)/E_1 + F'' \times 2pr(1 - \nu_2^2)/E_2 = F \times 2pr(1 - \nu_2^2)/E_2 \quad [3]$$

$$F = F' \times \{[E_2 \times (1 - \nu_1^2)]/[E_1 \times (1 - \nu_2^2)]\} + F'' \quad [4]$$

$$F' = 1 - [(1 + n_1^2 \, H^2/r^2)^{0,5} - n_1 \times (H/r)] \times \{1 + n_1 \times (H/r)/[2(1 - \nu_1)(1 + n_1^2 \, H^2/r^2)^{0,5}]\} \quad [5]$$

$$F'' = \left[\left(1 + H_{eq}^2/r^2\right)^{0,5} - H_{eq}/r\right] \times \left\{1 + \left(H_{eq}/r\right)/\left[2(1 - \nu_2)\left(1 + H_{eq}^2/r^2\right)^{0,5}\right]\right\} \quad [6]$$

Avesani Neto (2019) used coefficients n and n_1 unitary in his proposition. When both these coefficients are equal to 0.9 and the coefficients of Poisson are equal to 0.5, this formulation turns similar to that shown by Odemark (1949). Also, considering $n=n_1=1.0$, $v_1 = v_2 = 0.5$ and changing H by H_{eq} in Equation 6, it turns equal to Palmer and Barber's solution (Palmer and Barber, 1940).

Odemark (1949) adopted the values $n=n_1=0.9$ to obtain a better adjustment with the theoretical solution obtained by Burmister (1943) when both the materials are considered incompressible ($v=0.5$). When the material of the upper layer is concrete, the factor n adopted by the author was equal to 0.83 and, because of the high ratio of modules, his formulations will not depend on coefficient n_1. As regards this coefficient, Odemark justified that it is needed to correct the height of the upper layer to calculate its compression, considering all the mean with the same elastic properties of the first layer.

In recent works, Hirai (2007) adopted n and n_1 equal to 1 when $E_1/E_2 \geq 1.0$ and derived another formulation of the equivalent height when this relationship is lower than 1.0. Dhar and Tarefder (2011) developed a spreadsheet to estimate the displacement of a strip footing load by estimating the vertical and horizontal normal stresses and considering factor $n = (2/3)^{1/3} \approx 0.874$ for determining the equivalent thickness.

Concerning factor F, it multiplies the displacement obtained if the load is directly applied onto the soil of the subgrade. Considering that the subgrade still works with an elastic behavior, F is the inverse of the improvement factor I_f (Equation 7):

$$I_f = 1/F \qquad [7]$$

2. COMPARISON BETWEEN THE RIGOROUS (TE) AND THE APPROXIMATED (TET) METHOD PROPOSED BY AVESANI NETO (2019)

For this study, firstly parametric simulations of two-layer systems were carried out with a circular load, changing the relationship of the elastic modulus (E_1/E_2), the Poisson coefficients (v_1 and v_2) and the ratio of height H over the radius of load r. The software MnLayer (Khazanovich e Wang, 2007) was used to determine the elastic displacement obtained in the center of the plate for a unitary load (p=1). The values adopted for these simulations are shown in Table 1.

Table 1 - Adimensionals parameters adopted for this study

H/r	E_1/E_2	v_1, v_2
0.5;	1; 2; 5;	0.00; 0.10;
1.0; 1.5;	10; 20; 50;	0.20; 0.30;
2.0; 4.0	100; 200; 500	0.40; 0.49

With these displacements, factor F of Equation 4 was obtained by manipulating this into Equation 8:

$$F = s \times E_2/[2p \times r \times (1 - v_2^2)] \qquad [8]$$

These factors F are compared with the values obtained by Equations 2 to 6 of the TET method proposed by Avesani Neto (2019), adopting the values of n and n_1 equal to 1.0 and 0.9. As an example, Figure 2 shows these values adopting for pairs (v_1; v_2) as (0.20; 0.49), (0.20;0.40) and (0.20;0.20). By using the coefficients n and n_1 equal 0.9, less errors on the factor F are observed, by comparison with the use of n and n_1 equal 1.0. The tendency is the overestimation of F for high values of the ratio moduli E_1/E_2. When the moduli ratio is nearing to 1, these errors undervalue the factor F. Regarding the variation in the Poisson coefficients, these errors are smaller when considering that the subgrade has its coefficient v_2 equal to 0.49. The absolute errors obtained with $n=n_1$ adopted as 1.0 and 0.9 for each combination of v_1 and v_2 are shown in Table 2. The error was calculated by Equation 9.

$$error = |F_{TE} - F_{TET}|/F_{TE} \qquad [9]$$

Table 2 – Maximum absolute errors of value F calculated by TET as compaired with TE.

v_1	$n = n_1 = 1.0$						$n = n_1 = 0.9$					
	$v_2 =$ 0.00	$v_2 =$ 0.10	$v_2 =$ 0.20	$v_2 =$ 0.30	$v_2 =$ 0.40	$v_2 =$ 0.49	$v_2 =$ 0.00	$v_2 =$ 0.10	$v_2 =$ 0.20	$v_2 =$ 0.30	$v_2 =$ 0.40	$v_2 =$ 0.49
0	14.6%	15.6%	16.2%	15.9%	14.2%	10.1%	6.3%	6.6%	7.2%	6.9%	5.0%	3.8%
0.1	14.6%	15.6%	16.2%	15.9%	14.2%	10.1%	6.9%	6.7%	7.3%	7.0%	5.1%	4.1%
0.2	14.7%	15.7%	16.3%	16.0%	14.2%	10.2%	7.7%	6.7%	7.3%	7.0%	5.1%	4.1%
0.3	14.8%	15.8%	16.3%	16.1%	14.3%	10.2%	8.9%	6.8%	7.4%	7.1%	5.2%	3.6%
0.4	14.9%	15.9%	16.5%	16.2%	14.4%	10.4%	12.1%	9.0%	7.5%	7.2%	5.3%	2.4%
0.49	17.9%	16.1%	16.6%	16.3%	14.5%	10.6%	19.5%	15.6%	11.5%	7.5%	5.4%	2.4%

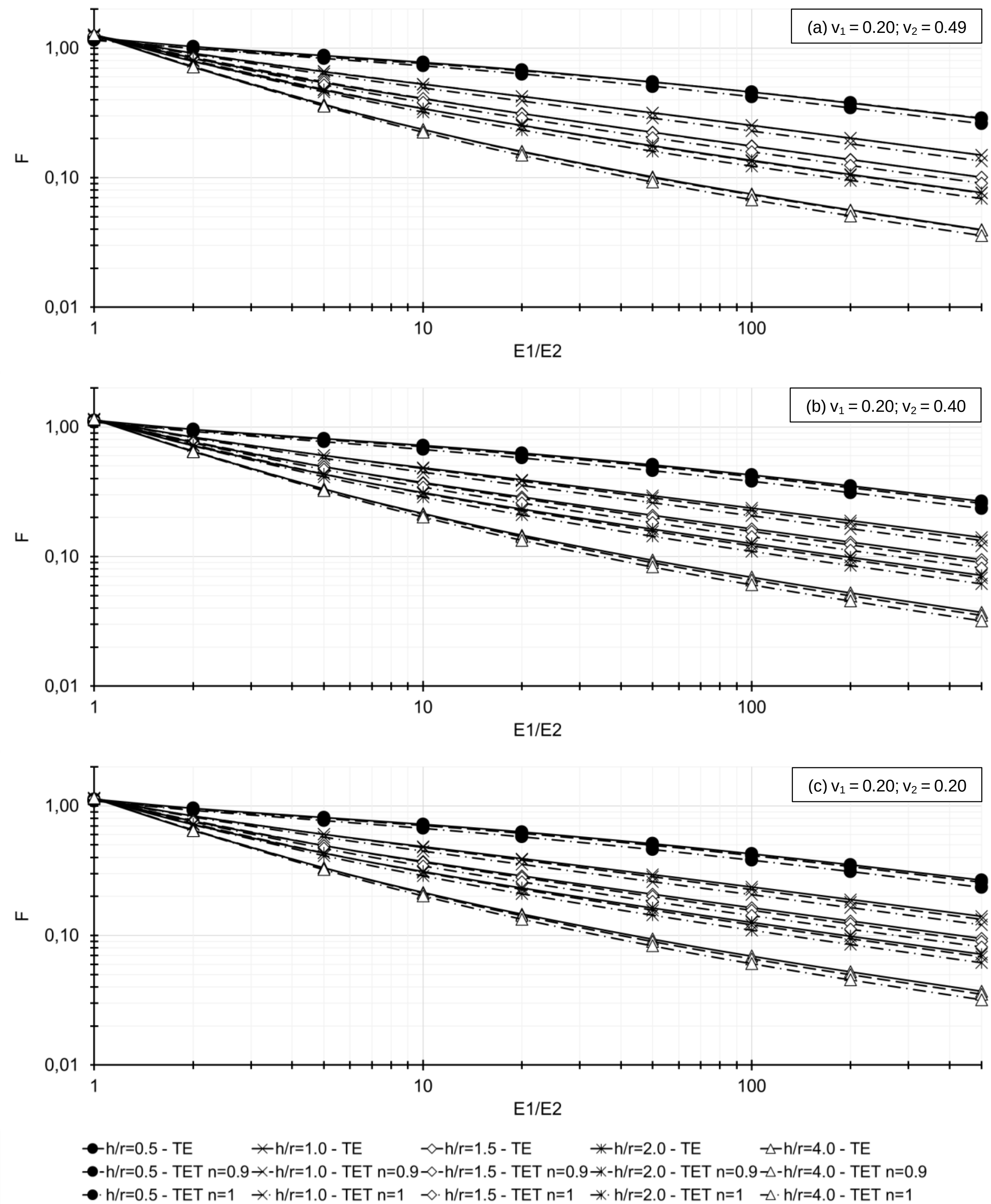
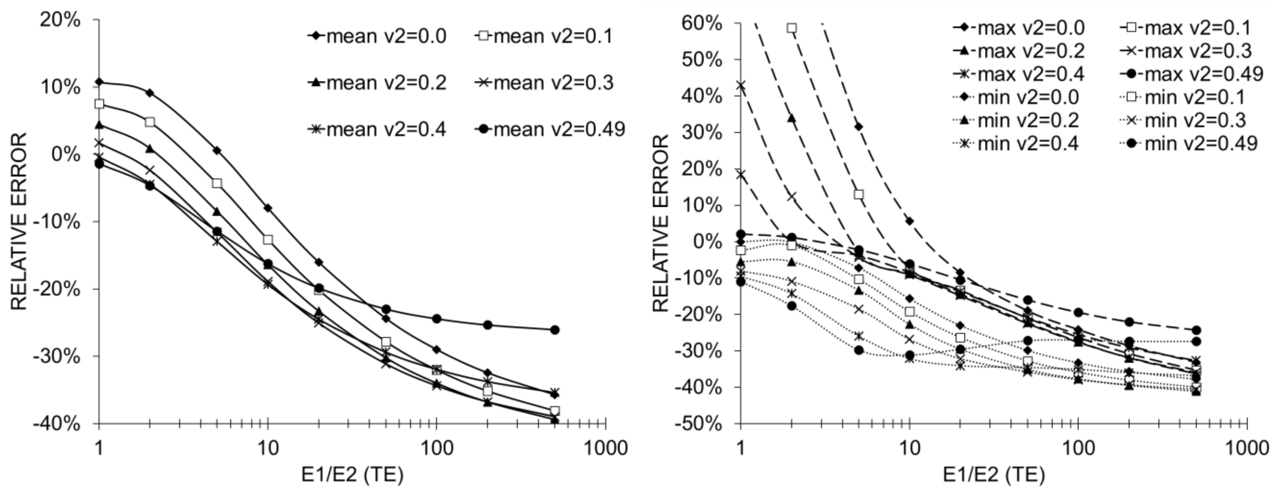


Figure 2 - Value of F by the TE and the TET

In the Equation 9, F_{TE} is the value F obtained by the simulation with the software MnLayer and F_{TET} this value found by the theory of equivalent thickness (Equations 2 to 6). These errors of the determination of F reflect on the back-calculation of the Young's modulus of the reinforced soil layer over the subgrade modulus. Adopting the coefficients of Poisson and knowing the geometric parameter H/r and elastic parameters of the subgrade for small displacements after analyzing the plate load test performed directly on this layer, the back analysis in this case consists in finding the E_1/E_2 which results in the same factor F. That was done by using the Solver from Microsoft Excel. The differences between the rigorous and the approximated method have a variability depending on the geometry of the problem, values of Poisson coefficients and of the moduli ratios; however, it is possible to characterize it by a mean value of relative error, a function of the ratio of modules. Figures 3 and 4 show the mean, the maximum and the minimum relative error of the ratio of modules E_1/E_2 obtained for $n=n_1$ adopted as 1.0 and 0.9 for each value of the Poisson coefficient of the subgrade. The relative error was determined by Equation 10:

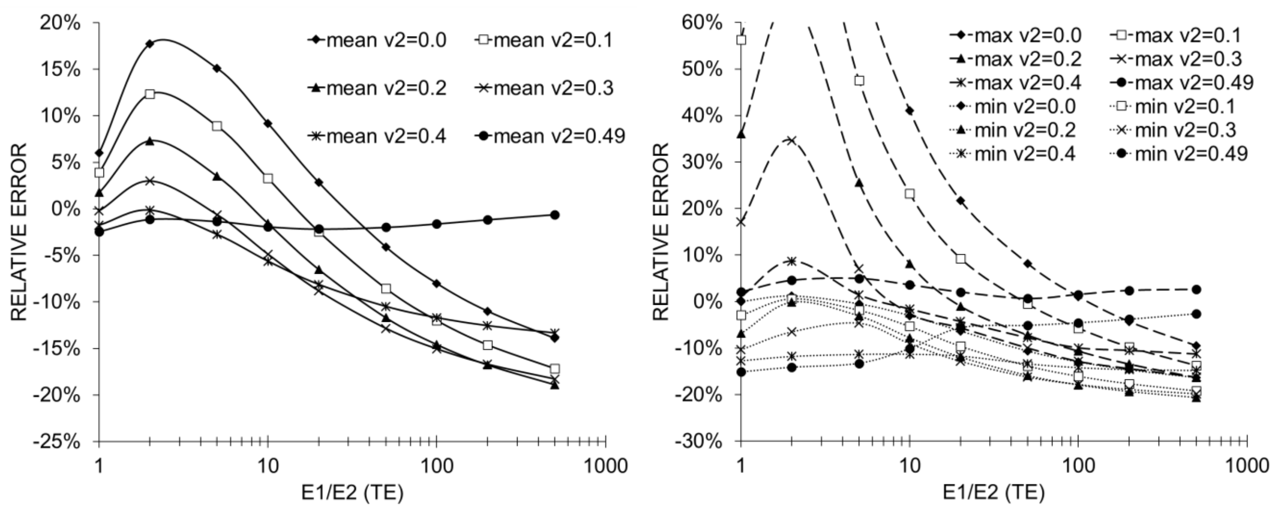
$$error = [(E_1/E_2)_{TET} - (E_1/E_2)_{TE}]/(E_1/E_2)_{TE} \qquad [10]$$

Positive errors indicate that the TET formulation overestimates the value of the modulus of elasticity of the superior layer. The highest positive value of the maximum error in the low values of E_1/E_2 occur for the geometry with H/r = 0.5, reaching over 130% when $v_2 = 0.0$. Therefore, for other geometries, this error is less than 31% for all the combinations of Poisson coefficients.



(a) Mean (b) Maximum and Minimum

Figure 3 - Relative errors of the estimated ratio E1/E2, for $n = n_1 = 1.0$



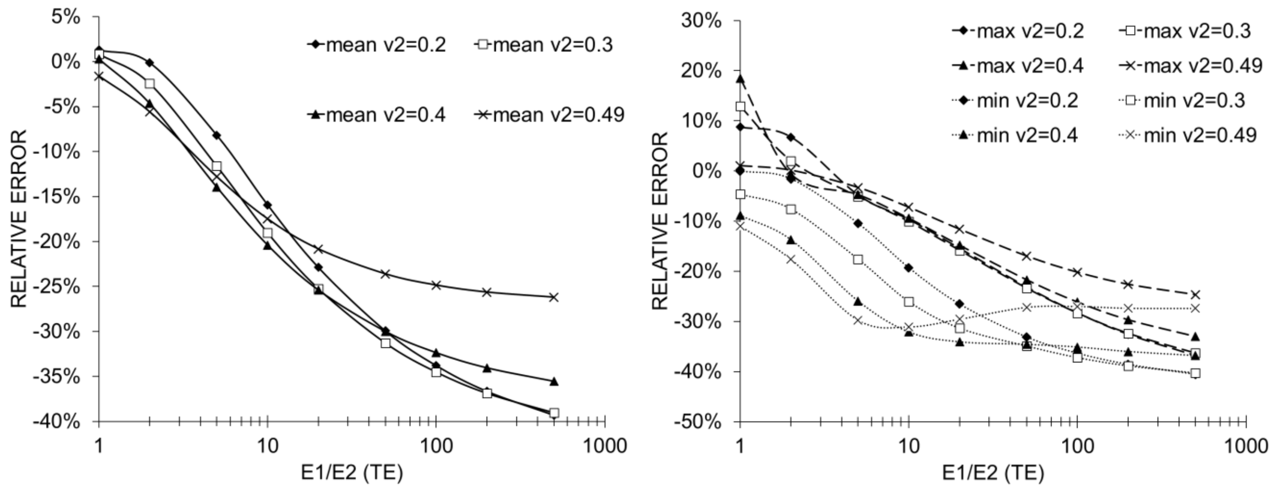
(a) Mean (b) Maximum and Minimum

Figure 4 - Relative errors of the estimated ratio E_1/E_2, for $n = n_1 = 0.9$

However, the whole range of the coefficients of Poisson is higher than the values of the soils involved. If the analysis contained only the pairs of v_1 and v_2 highlighted in Table 3, the maximum and the minimum overvaluation of the back-calculated modulus of the superior layer would be 18.45% and -40.52% for n = 1.0 and 20.12% and -19.94% for n = 0.9. These values are shown in Figures 5 and 6.

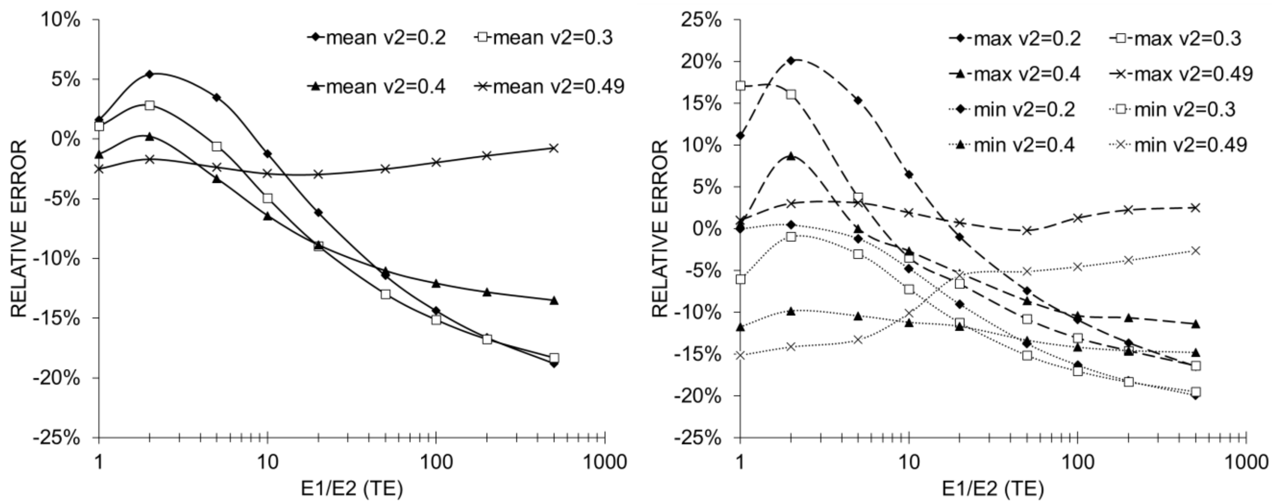
Table 3 - Values of Poisson coefficients considered for optimizing the TET

Reinforced Layer → v_1	Subgrade → v_2 0.00	0.10	0.20	0.30	0.40	0.49
0.00						
0.10						
0.20						
0.30						
0.40						
0.49						



(a) Mean (b) Maximum and Minimum

Figure 5 - Relative errors of the estimated ratio E_1/E_2, for $n = n_1 = 1.0$ and (v_1, v_2) of Table 3



(a) Mean (b) Maximum and Minimum

Figure 6 - Relative errors of the estimated ratio E_1/E_2, for $n = n_1 = 0.9$ and (v_1, v_2) of the Table 3

3. SEARCH FOR AN N WHICH REDUCES THE MAXIMAL ERROR

3.1 Factor $n = n_1$ constant

It is possible to imagine that there is a combination of n and n_1 in which the error of the ratio of modulus of the analyzed layer and the subgrade is minimized. In turn, the simplicity of the back-calculation method using the TET could be lost on an attempt to search a perfect match between the rigorous method, using the TE. Therefore, in this study, there was an effort to find a better value of $n=n_1$ that best agreed with these results, focusing in values of Poisson coefficients interested in the back analysis of a plate load test on geocell-reinforced soil. The combinations of pairs of values (v_1; v_2) are already shown in Table 3.

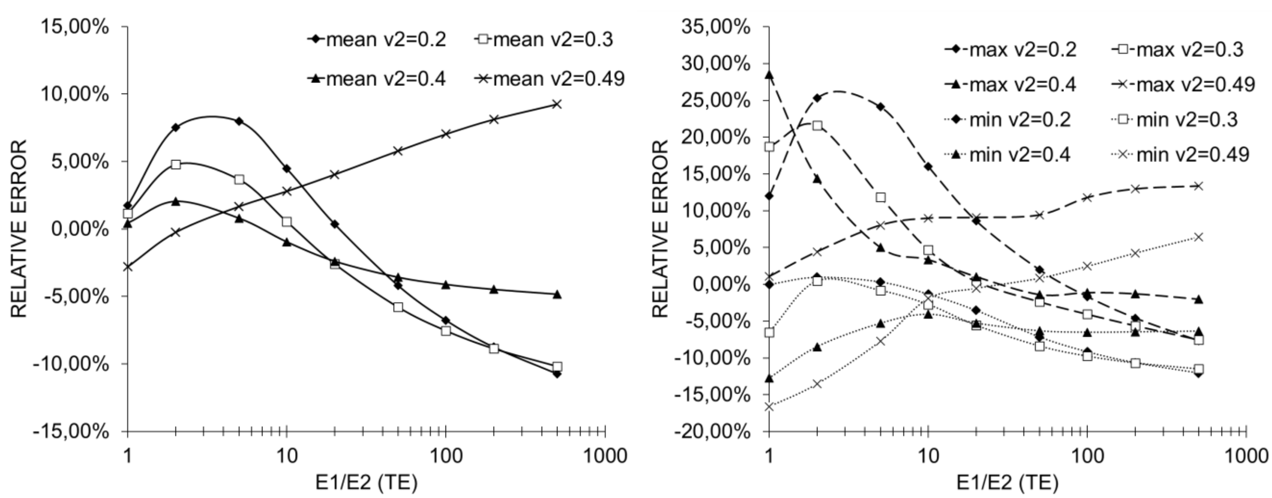
Utilizing the Solver of the Microsoft Excel, the value obtained of n and n_1, which minimizes the maximum relative error of the value of factor F for the pairs of coefficients of Poisson considered, was approximately 0.870 the error of F was less than 4.41% (against 16.3% and 7.4% for n equal to 1.0 and 0.9). Regarding all the range of values of Poisson coefficients, the mean error for the highest value of moduli ratio E_1/E_2 studied is reduced to about 13%. However, for low values of E_1/E_2 and for thin reinforced layers (H/r = 0.50), the mean error achieves a value of 20% and the maximum error obtained reaches 200%. If only the pairs of (v_1; v_2) highlighted in Table 3 are considered, the maximum overvaluation of the moduli ratio is reduced to about 28.5%. Table 4 shows the maximum absolute error of F and Figure 7 shows the relative errors of the estimation of the Young's modulus of the reinforced layer.

Table 4 – Maximum absolute error of F for n ≈ 0.870

v_1	v_2					
	0.00	0.10	0.20	0.30	0.40	0.49
0	7.8%	4.9%	4.2%	3.9%	4.1%	4.2%
0.1	8.2%	5.5%	4.3%	4.0%	3.9%	4.5%
0.2	9.1%	6.1%	4.3%	4.0%	3.3%	4.4%
0.3	10.2%	7.0%	4.4%	4.1%	2.2%	3.9%
0.4	13.0%	9.7%	6.4%	4.2%	2.4%	3.9%
0.49	20.0%	16.0%	11.9%	7.7%	3.6%	3.9%

3.1 Factors n and n_1 changing with the Poisson coefficient

There is the possibility of considering factor n and n_1 as a function of the Poisson coefficient of the layers. For this, a search of a parabolic curve in which the best adjustment for determining F was conducted, adopting $n= n_1$ as a function of v_2. These values were searched using Solver, adopting a parabolic function which minimizes the max error obtained to calculate F. Table 5 lists the functions that have the best fit that minimizes the maximum error of the pairs of coefficients indicated in Table 6. With these values, the maximum error obtained for factor F was 4.19%. The impact of the back-calculation of Young's module is shown in Figure 8.



(a) Mean (b) Maximum and Minimum

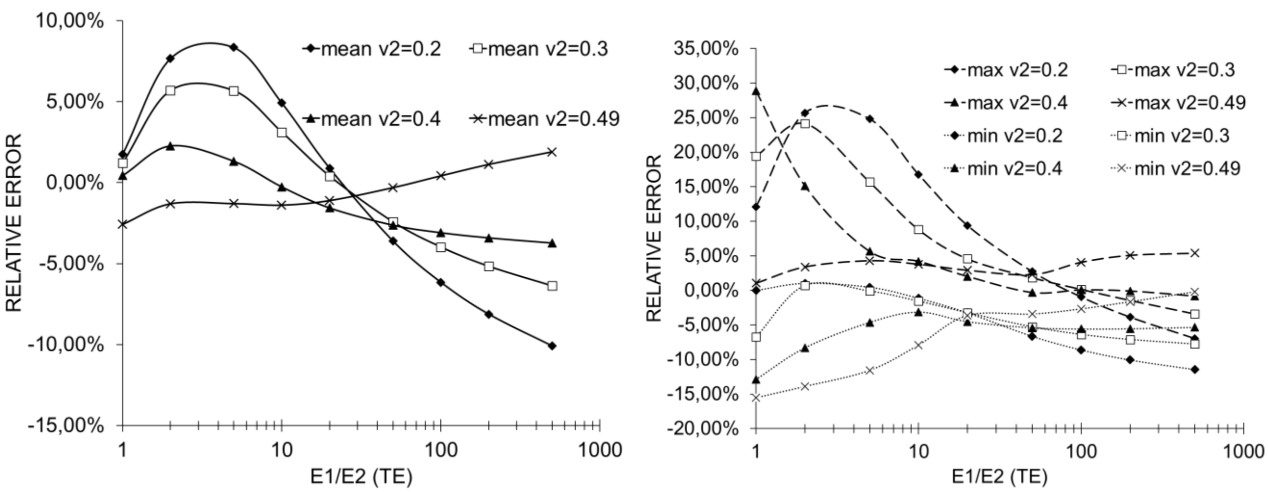
Figure 7 - Relative errors of the estimated ratio E_1/E_2, for $n = n_1 \approx 0.870$

Table 5– Values of $n=n_1$ varying with the Poisson coefficient of the subgrade

v_2	0.20	0.30	0.40	0.49
$n=n_1$	0.868	0.857	0.867	0.892
Function	$n = 0.984\upsilon_2^2 - 0.5972\upsilon_2 + 0.9481$			

Table 6 – Maximum absolute error of F for $n = n_1 = f(v_2)$

v1	v2			
	0.20	0.30	0.40	0.49
0.20	4.10%	-2.80%	3.32%	4.19%
0.30	-4.19%	-2.77%	2.13%	3.70%
0.40		-3.43%	2.04%	2.39%
0.49			-3.61%	1.82%



(a) Mean (b) Maximum and Minimum

Figure 8 - Relative errors of the estimated ratio E_1/E_2, for $n = n_1$ varying with v_2

4. CONCLUSIONS AND FINAL CONSIDERATIONS

We here present our investigation into the accuracy of the method for considering the TET when using it for the back-calculation of two-layer soil structures subjected to a plate load test, usually employed to evaluate the improvement of geocell reinforcement systems, and to research a better value for coefficients n and n_1 to enhance the fit between the analysis with the rigorous method derived by the Theory of Elasticity and the approximate method.

Considering the classical values of n and n_1, the best fit is achieved if these factors are equal to 0.9, for obtaining factor F and for the back-calculation. Observing the pairs of Poisson coefficient interested for the analysis of the soil reinforced with geocell shown in Table 3, the mean relative error found is an underestimation of the moduli of 5.90%. For n equal 1, this value is 20.4%. It shows that the use of a number smaller rise the value of the estimation. It is evidenced on low values of E_1/E_2. Furthermore, the amplitude of the error (i.e., difference between the maximum overvaluation and undervaluation) is increased. For example, for values of E_1/E_2 between 1 and 5, the amplitude was going from 29.4% (to n=1.0) to 34.2% (n=0.9), even with a minor error of the direct calculation of F (respectively, -16.3% and -7.4%).

Regarding the search for a better value of n to minimize the relative error of factor F, the best constant value for n and n_1 that reduces the error of the calculation of F is 0.870, with a relative error of 4.41%. This value is near the value obtained and used by Dhar and Tarefder (2011). The mean error obtained on the back-calculation of the ratio of moduli is -0.14% and the amplitude of errors obtained is 45.1% on the same range of values of E_1/E_2 previously cited. For factor n as a function of v_2, these values were -0.64% and 44.4%. It shows that the use of a changing with the Poisson coefficient with a parabolic curve isn't effective to enhance the method, compared with a constant value. Despite this result, this variation of n with v_2 is interesting, mainly considering the incompressible subgrade (i.e., $v_2 \approx 0.5$, with $n = n_1 = 0.892$), when the mean error found is -0.50% and range of error was between -15.5% and 5.4% (against 3.96%, -16.6% and 13.4% of $n=n_1=0.870$). That value of n is near to the value adopted by Odemark (1949). By comparison, the mean, minimum and maximal relative error when $n=n_1=0.90$ is used are -2.1%, -15.1% and 3.1%.

To conclude, we show that the TET could be used for the back analysis of the elastic parameters of the reinforced soil with geocells subjected to plate load tests and modeled as a two-layer system soil structure in an approximate way.

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