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Reliability analysis of internal and external stability of geosynthetics reinforced soil retaining walls

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ABSTRACT
In the traditional design of geosynthetic reinforced soil walls (GRSW), allowable stress design (ASD) and semi-probabilistic design (SPD) approaches are used to determine the factors of safety against the failure mechanisms. However, these approaches cannot explicitly consider the uncertainties in the design process, especially geotechnical uncertainties which are typically project specific. The present paper introduces a framework for the reliability design of GRSW to explicitly address uncertainties in the design process and account for the actual safety and reliability level of a given design. The basics of probabilistic analysis and design for the internal and external stability limit states of GRSW are explained, and reliability analyses of GRSW are put in a rational framework, where concepts are general and can be applied to any GRSW for which the stability can be expressed by limit state functions, even if the present paper addresses only vertical walls. The limit state functions for the five failure mechanisms of internal and external stability are defined, and are used to calculate margins of safety in terms of probability of failure through Montecarlo simulations, where all parameters can be set as either deterministic (with no associated variability) or probabilistic (with associated variability). The probabilistic analyses can be carried out repeatedly by changing the deterministic and the stochastic parameters, and/or their associated variability. Results can provide a useful decision-making tool for preliminary design of GRSW based on target reliability levels. An example is presented to demonstrate the significance of the proposed framework.

1. INTRODUCTION

Geosynthetic reinforced soil walls (GRSW) are more and more specified in geotechnical engineering projects all around the world thanks to the redundancy of stability mechanisms and their ductile performance against various loading and foundation deformation. According to Zannoni (2016), while GRSW are widely used in geotechnical engineering, the performance prediction from a design model can be highly uncertain because of the difficulties in accurately determining geotechnical and loading parameters in the design. Failure to consider such uncertainties could lead to either expensive over-design or under-design, which may prolong or reduce the construction period and in the worst case scenario may fail to meet the performance requirements. Usually loading conditions are known quite well to a certain extent, hence the challenge is usually to define the geotechnical properties of the soils involved, either in situ soils (typically the soil at the back of the GRSW body and the foundation soil) or imported soils (typically the GRSW fill).

The design of GRSW is necessarily based on the geotechnical properties of the soils involved: depending on the size and importance of the project, and the geology of the site, the geotechnical investigation might vary in methodology and number of in-situ and laboratory tests to determine, to a certain degree of confidence, the soil properties.

Working stress and now partial factor design based codes have the challenging job to try ensuring the safety of the design within a certain degree of safety, based usually on the type of structure.

What is usually very difficult and requires experience and knowledge is setting the correct values of the geotechnical parameters of soils: what is the right value of friction angle or cohesion for the actual soil strata? Even if the design is formally correct, when these values are incorrect, the whole design can be compromised. On the other hand, geosynthetics are manmade manufactured products with very high degree of confidence, since all properties are defined as Minimum Average Roll Values (MARV) or 95 % lower confidence limit values, due to the industrial manufacturing which allows many tests to be performed on the same product under the same conditions.

The aim of the present paper is to demonstrate that high confidence in the design of a GRSW can be obtained even with high variability of soil parameters, consequent to poor/inexpensive geotechnical investigation.

In this paper the effects of uncertainties associated with geotechnical and loading parameters on the performance of GRSW are evaluated using advanced reliability methods for assessing failure probabilities for the relevant failure modes. This reliability approach accounts for the stochastic nature of geotechnical and geosynthetics parameters and provides useful information on the level of design performance under uncertainties. An example is presented to demonstrate the effectiveness of the proposed framework. The proposed reliability approach provides a useful tool for the engineer to make a more informed design decision based on the target reliability levels.

2. VARIABILITY OF PARAMETERS OF SOILS AND GEOSYNTHETICS

It is known that any geotechnical engineer evaluating a geotechnical investigation will determine different design values based on their knowledge, experience with the testing method, the site, the geology and the operators (Bond and harris, 2008). In EN 1990 (2002) the engineer is required to define the characteristic value (X_c) of materials, including soil: when a low value of material or product property is unfavorable, X_c should be defined as the 5 % fractile value; when a high value of material or product property is unfavorable, X_c should be defined as the 95 % fractile value. This definition works well with man-made materials such as concrete or geosynthetics, however for soil this might not apply since some parameters might have a coefficient of variation up to 60 % (Phoon 1995). For this reason in EN 1997-2 (2007), dealing with geotechnical designs, the characteristic value is defined as “cautious estimate”, which is open to any interpretation.
A further complication is the volume of soil involved in the design. If it is agreed that soil is a variable material in its performance, indeed it will vary spatially (its variation in time is not considered in this paper).
To calculate the pile tip resistance a small amount of soil is considered, and it is well defined as the depth is known; whereas for a GRSW stability the soil that should be considered includes the wall fill, the back soil and the foundation soil, which might means thousands m^3 of soil strata with different history and behaviour.
In a geotechnical investigation executed in South Africa (Parrock 2014), 17 tests were undertaken on the same material. The density of the soil is easy to measure (Coefficient of Variation CV was 3.5 %); the grading highlights the difficulties of using the hydrometer method (CV was 55 % for the 0.002 mm fraction); less variability was got using the wet sieving for the granular part of the grading (CV was 24 %). Following the definition of characteristic value, the X_c for the cohesion and friction angle at 95 % confidence limit would have been a cohesion of 0 and a friction angle of 18°. While for the cohesion there is a good agreement, for friction angle many interpretations might have suggested to be over conservative. Thus, the Eurocode 7 (EN 1997-2, 2007) amends the definition of characteristic value to “a cautious estimate of the value affecting the occurrence of the limit state”, thus leaving the choice to the designer based on the category of the structure.
The improve the definition of the characteristic values the designer should have repeated correlated values. In a simple example, the amount of data required for a slope stability analysis considering only one uniform and homogenous soil (which is already a simplification) would require to get more than 5 samples of friction angle, cohesion and soil density for which the variance is known. Only in some instances, where the magnitude of the project allows (i.e. power stations, airports, industrial areas), with a thorough geotechnical investigation the amount of data might allow the designer to get a good estimate of the characteristic values with very small CV.
Geosynthetics materials are man-made materials manufactured under strict quality control procedures in order to ensure the highest confidence level of the required properties. Depending on the function and application of the geosynthetics, different properties are required; to be in line with the subject of this paper, here only soil reinforcement application will be considered.
According to EN 13251 (2002) the properties required for a geosynthetics to fulfill the reinforcement function are: tensile strength and elongation at maximum load. Mean and standard deviation shall be reported for all properties.
It should be noted that these characteristics are not exhaustive to thoroughly assess the behaviour of a geosynthetics for soil reinforcement since no long term properties, installation damage and durability are specified in EN 13251 (2002).
The above properties are part of the Reduction Factor RF specified in ISO TR 20432 (2007), which was developed to ensure the compatibility of geosynthetics for the soil reinforcement function.
As tensile strength is a function of time, set by the design life of the structure, Reduction Factors RF are required for tensile creep and chemical degradation; both these RF are based on extrapolation methods which takes into account the uncertainties in extrapolation over long durations: uncertainties of creep testing and uncertainties of accelerated chemical tests. The ability to perform repeated testing in a controlled environment has enabled geosynthetics to be characterized by a coefficient of variation less than 10 % at 120 years design life.
A commercial bonded polyester geogrid (Linear Composites, 2015), made up of a regular array of composite geosynthetic straps, nominally interconnected laterally to form a geogrid with high unidirectional strength is characterized by the high reliability of geosynthetic properties, both in short term (CV = 1.17 % for the tensile strength) as well as in the long term (the Partial material factor applied to reinforcement, f_m, is just 1.05).

3. RELIABILITY APPROACH TO GRSW DESIGN

Several papers has been published on a reliability approach to GRSW design, including Bathurst, R. J. (2018), Chalermyanont, T. and Benson, C.H. (2004), Chen et al (2016), Huang, B. and Bathurst, R. J. (2009), Sayed et al (2008), Yang et al (2010), Zannoni, E. (2016), and other papers.
According to Zannoni (2016), the current design philosophy for GRSW is based on limit state analyses applying partial factors. This design approach enables the study of the behaviour of the structure at different states (usually ultimate limit states and serviceability limit states) and it allows to apply reduction factors to the single parameters rather than one factor of safety for all (working stress approach).
The scope of any design philosophy is to ensure that the design is safe, but how much is it safe? This is a hidden issue as a temporary structure should not be characterized by the same margin of safety that a bridge abutment requires.

Most design guidelines introduce a classification of structure at the beginning, like SANS 10160-5 (2010) and BS 8006-1 (2010), usually based on the design life and the complexity of the structure itself.

The idea behind structure category is to apply a further partial factor to the overall design which increase the reliability of the design itself, thus from a reliability index (see Eq. 1) of 3 (US Army Corps of Engineers, 1997) for a temporary retaining wall, multiplying by the partial factor of 1.5 the reliability index increases to 4.5. Since reliability is linked to probability (Eq. 1), a reliability index of 3 generally means a probability of failure of approx. $1x10^{-3}$, while a reliability index of 4.5 generally means a probability of failure of approx. $5x10^{-6}$ (Figure 1). Bathurst (2018) argues that the internal stability design for MSE walls in the UK is based on a partial factor approach in which factors are applied to soil and reinforcement material properties and to load contributions in different combinations to ensure safe designs. Geotechnical foundation design codes in North America adopt a load and resistance factor design (LRFD) approach which has been used by structural engineers for decades in Canada and the USA. In this approach, load terms are multiplied by load factors (magnitude of one or more) and the resistances are multiplied by a single resistance factor (with a magnitude of one or less). The intent of a properly calibrated limit state design equation expressed in a LRFD framework is to ensure that a target maximum probability of failure will not be exceeded. However, the load and resistance factors that appear in LRFD codes have been selected largely by fitting to factors of safety used in allowable (working) stress design (ASD) past practice. Whether a designer uses a partial factor approach as in the UK or a LRFD approach as in North America, the margin of safety expressed probabilistically is unknown. This leads to the conundrum of a limit state being satisfactory when viewed from a factor of safety point of view but unsatisfactory from a probability of failure perspective. This point is demonstrated for the case of the rupture and pullout limit states for the geosynthetic reinforcement layers in the MSE wall in Fig. 2.Left (Bathurst, 2018). The nominal tensile load Q_n can be calculated using one of a number load models found in the literature and design codes. Similarly, the nominal resistance R_n can be calculated for tensile rupture and pullout limit states using equations found in design guidelines. In conventional allowable stress design the ratio of nominal resistance to nominal load defines the factor of safety; the resistance term is adjusted so that the factor of safety satisfies a minimum acceptable value. However, both nominal resistance and nominal load sides have uncertainty, measured by their covariance COV, as visualized by the idealized frequency distributions in Figure 2.Right (Bathurst 2018). Notionally, the area of the overlap of the lower tail of the resistance distribution on the right with the upper tail of the load distribution on the left indicates a non-zero probability of failure. Clearly, two different combinations of load and resistance distributions can have different probabilities of failure (P_f) while the average factor of safety remains the same.

An alternative parameter to quantify margins of safety in geotechnical engineering is the reliability index β. The relationship between probability of failure P_f and reliability index β is:

$$P_f = 1 - F(\beta) \qquad (1)$$

where F is the standard normal cumulative distribution function, CDF (NORM.DIST in Excel).

Bathurst (2018) argues that values of β = 2.33 and 3.09 correspond to probabilities of failure of 1/100 and 1/1000, respectively. The smaller β value is recommended as the target minimum reliability index for internal limits states design and LRFD calibration for GRSW. This value may appear small but GRSW walls are highly strength-redundant systems. In other words, if one reinforcement layer fails, other layers can compensate and thus system failure is unlikely. If a reinforcement layer is designed to just satisfy a target reliability of β = 2.33, the corresponding factor of safety can be as high as 1.70.

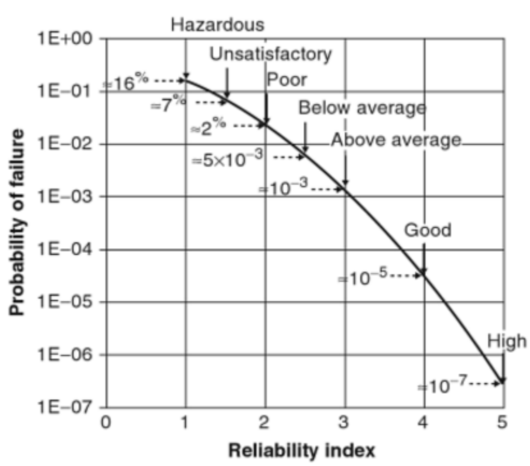

Figure 1. Probability of failure vs reliability index (from US Army Corps of Engineers, 1997)

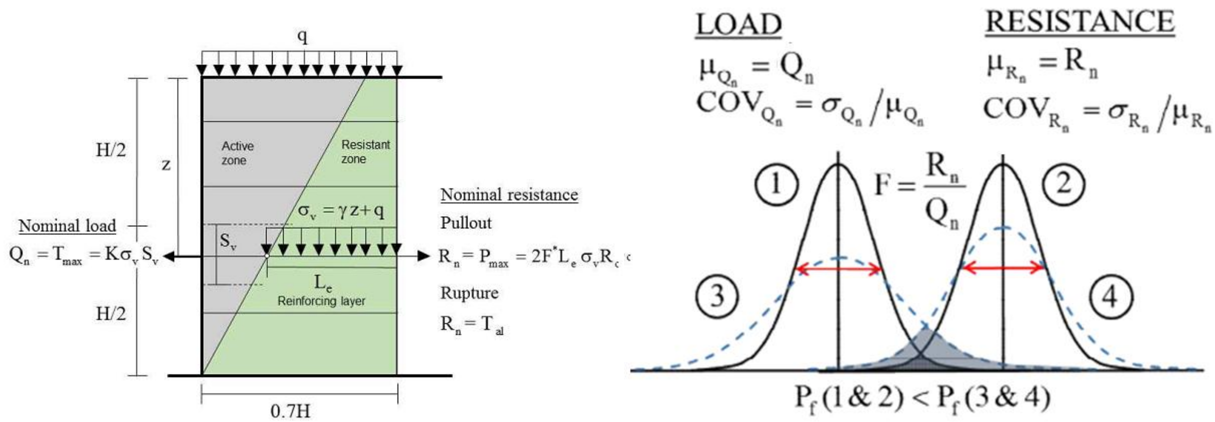

Figure 2. Left: Nominal load and resistance equations for reinforcement rupture and pullout limit states for internal stability of GRSW; Right: Factor of safety and probability of failure concepts (modified from Bathurst 2018).

4. PROBABILISTIC MODEL OF GEOSYNTHETIC REINFORCED SOIL WALLS

There are mainly three types of stability requirements for the deterministic analysis of GRSW, including the external stability, the internal stability, and the global stability. External stability concerns about the stability of the entire reinforced soil wall body, including checking for the possible failure modes such as sliding, overturning, and bearing capacity failures. The internal stability concerns about the stability of the reinforced materials, including checking for the possible failure modes due to pullout and tensile failure.
In the present paper only limit states for internal and external stability are considered. For considering global stability, additional equations would be required, which are beyond the scope of the present paper.
The limit state functions G(x) which can be derived to evaluate the performance of GRSW against each failure mode for external and internal stability are represented by the equations providing the difference of resisting and active forces or moments for each of the considered failure modes: G(x) = (R – A), where R represents the resisting forces or moments, and A represents the active forces or moments.
In the present paper these limit functions are set for the design of GRSW in uniform granular soils with zero effective cohesion, but they can be easily extended to the case of cohesive soils.
The framework for defining the limit state functions is hereinafter presented, based on the resisting and active forces or moments for each failure modes. The schemes for each failure mode are presented in Fig. 3, where uniform vertical spacing S for all reinforcement layers is assumed.
For sake of brevity only geogrids will be referenced, but the same concepts apply to any geosynthetic reinforcement.

Sliding Failure
For GRSW the critical sliding usually occurs along the geosynthetic reinforcement at base, when the friction between the fill and the geosynthetic is not enough to compensate for the external load, which causes the retaining wall to slide. With reference to the scheme in Fig. 3.a, the limit state function for the sliding failure is:

$$G_{ds} = L \cdot f_{ds} \cdot \tan\varphi_f \cdot (\gamma_R \cdot H + q) - (0.5 \cdot \gamma_s \cdot H^2 + q \cdot H) \cdot \tan^2(45 - \varphi_s/2) \quad (2)$$

where:
φ_f, φ_s = friction angle of foundation soil and back soil, respectively (deg)
γ_R, γ_s = unit weight of reinforced soil and back soil, respectively (kN/m^3)
f_{ds} = direct shear factor (-)
H = height of wall (m)
L = length of reinforcement (m)
q = uniformly distributed surcharge (kPa).
Note that tan φ_f is used in Eq. (2) assuming that the friction angle of the foundation soil, φ_f, is lower than the friction angle of the reinforced soil, φ_R.

Overturning Failure
Overturning occurs when the soil thrust behind the GRSW body is great enough to offset the retaining wall by rotation around the wall toe. With reference to the scheme in Fig. 3.b, the limit state function for overturning is given by the difference of resisting and active overturning moments around the toe:

$$G_{ot} = 0.5 \cdot L \cdot (\gamma_R \cdot L \cdot H + q \cdot L) - (1/6 \cdot \gamma_s \cdot H^3 + 0.5 \cdot q \cdot H^2) \cdot \tan^2(45 - \varphi_s/2) \quad (3)$$

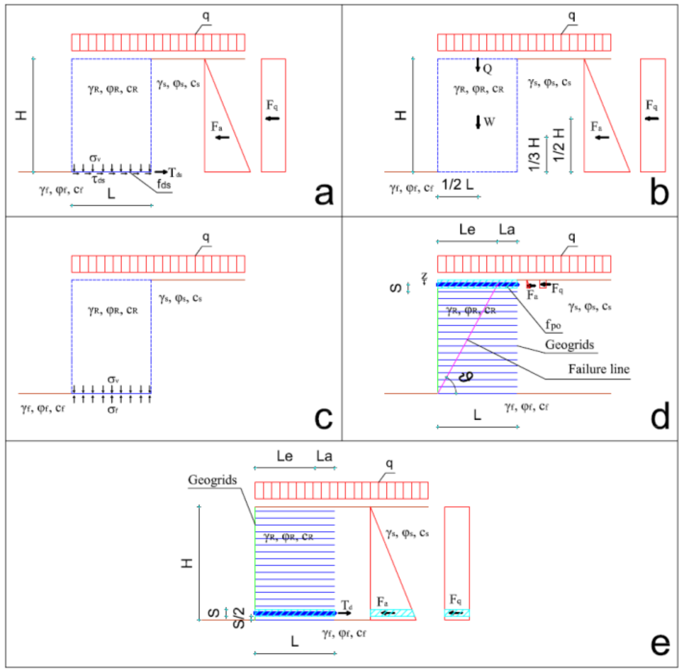

Figure 3. Schemes of the limit states for internal and external stability of GRSW: a. sliding; b. overturning; c. bearing capacity; d. pullout; e. tensile failure

Bearing Capacity Failure

Bearing capacity failure occurs when the subgrade soil beneath the reinforced soil wall fails under shear due to overloading or insufficiently constructed subgrades. With reference to the scheme in Fig. 3.c and using the Terzaghi formula for general shear failure of foundations, the limit state function for bearing capacity is given by the difference of the bearing capacity of foundation soil and the vertical stress on the foundation:

$$G_{bc} = 0.5 \cdot \gamma_f \cdot L \cdot N_\gamma - (\gamma_R \cdot H + q) \tag{4}$$

with:

$$N_\gamma = 2 \cdot \tan[e^{\pi \cdot \tan \varphi_f} \cdot \tan^2(45 + \varphi_f/2) + 1] \tag{5}$$

where:
φ_f = friction angle of foundation soil (deg)
γ_f = unit weight of foundation soil (kN/m^3)
N_γ = bearing capacity factor (-)

Note that the Terzaghi formula can be replaced by more complex formulas (Meyerhoff, Brinch-Hansen, etc.) in the limit state function (4), while the framework remains valid.

Pullout Failure

Pullout failure occurs when the geogrid do not have sufficient length to resist the soil thrust in the influence area of each layer, thus causing failure by pullout.

With reference to the scheme in Fig. 3.d, the critical condition usually occurs for the top geogrid, which has the lowest vertical stress producing the pullout shear stresses needed for anchorage of the geogrid itself in the fill behind the potential failure surface. The anchorage length L_a is defined as the total geogrid length L minus the length L_e from the face to the point where the potential failure surface intersects the geogrid. Assuming that the geogrid is an extensible reinforcement, the failure surface coincides with the Rankine failure surface, identified by a line passing from the toe and inclined of the angle ϑ. The limit state function for pullout is therefore given by the difference of the pullout resisting force developed along the anchorage length of the top geogrid and the soil thrust in the influence area of the top geogrid:

$$G_{po} = F_{po} - (F_{a-po} + F_{q-po}) \tag{6}$$

with:

$$F_{po} = 2 \cdot \tau_{po} \cdot L_a = 2 \cdot \tau_{po} \cdot (L - L_e) = 2 \cdot f_{po} \cdot (\gamma_R \cdot z + q) \cdot (L - (H - z))/\tan \vartheta \tag{7}$$

$$F_{a-po} = 0.5 \cdot \gamma_R \cdot \tan^2(45 - \varphi_R/2) \cdot (z + S/2)^2 \tag{8}$$

$$F_{q-po} = q \cdot (z + S/2) \cdot \tan^2(45 - \varphi_R/2) \quad (9)$$

$$\vartheta = 45 + \varphi_R/2 \quad (10)$$

$$z = H - (N_{GG} - 1) \cdot S \quad (11)$$

where:
F_{po} = pullout resisting force (kN/m)
$F_{a\text{-}po}$ = active horizontal force produced by the self weight of the fill soil for pullout (kN/m)
$F_{q\text{-}po}$ = active horizontal force produced by the surcharge for pullout (kN/m)
τ_{po} = pullout shear stresses on both sides (top and bottom) of the geogrid (kPa)
f_{po} = pullout factor (-)
φ_R = friction angle of reinforced soil (deg)
L = total length of the geogrid (m)
L_a = anchorage length of the geogrid (m)
L_e = length of geogrid between the face and the failure surface (m)
ϑ = inclination of the failure surface on the horizontal (deg)
z = depth of the geogrid from the top of wall (m)
N_{GG} = total number of geogrid layers (-)
S = uniform vertical spacing of geogrids (m).

Note that the variable surcharge q is both favorable for safety, when calculating F_q, and unfavorable, when calculating F_{po}: hence, strictly speaking, G_{po} should be calculated with both applying and not applying q. Nevertheless, the situation with q applied is usually the critical one.

Tensile Failure
Tensile occurs when the tensile strength of the geogrids is not enough to withstand the forces applied by the thrust of the soil. With reference to the scheme in Fig. 3.e, for GRSW the critical condition usually occurs for the first geogrid above the toe, which has to withstand the highest horizontal stresses multiplied by its influence area. The limit state function for tensile failure is therefore given by the difference of the design strength T_D of this geogrid and the active horizontal force produced by the fill soil and the surcharge on the influence area of the geogrid:

$$G_{tf} = T_D - (F_{a-ts} + F_{q-ts}) \quad (12)$$

with:

$$T_D = \frac{T_{ult}}{RF_{cr} \cdot RF_{id} \cdot RF_c \cdot RF_b} \quad (13)$$

$$F_{a-ts} = 0.5 \cdot \tan^2(45 - \varphi_R/2) \cdot [(H - S/2)^2 - (H - 3/2 \cdot S)^2] \quad (14)$$

$$F_{q-ts} = q \cdot S \cdot \tan^2(45 - \varphi_R/2) \quad (15)$$

where:
T_D = design tensile strength of the geogrid (kN/m)
$F_{a\text{-}tf}$ = active horizontal force produced by the self-weight of the reinforced soil for tensile failure (kN/m)
$F_{q\text{-}tf}$ = active horizontal force produced by the surcharge for tensile failure (kN/m)
RF_{cr} = Reduction Factor for tensile creep of geogrids (-)
RF_{id} = Reduction Factor for installation damage of geogrids (-)
RF_c = Reduction Factor for chemical damage of geogrids (-)
RF_b = Reduction Factor for biological damage of geogrids (-)

Note that RF_b can be assumed always equal to 1.0 for geosynthetic reinforcement; hence this parameter can be considered as a deterministic value, while the other RFs can have a variability in respect of their nominal value and therefore these are considered as stochastic values.
Here all uncertainty in the true magnitude of load and resistance terms for the limit states introduced above is solely due to the estimation of the friction angle (φ) and unit weight (γ) of soils, and of the design tensile strength of geogrids T_D; nevertheless, all parameters in the above defined limit state functions could be considered as stochastic variables.
Then the probability of failure of each limit state can be computed using the above limit state functions, by producing N sets of values using Monte Carlo simulations.
Specifically, random values of φ_R, φ_s, φ_f, γ_R, γ_s, γ_f, T_{ult}, RF_{cr}, RF_{id}, RF_c, are generated from probability distributions for these parameters within their set variabilities (in % of the nominal values) for a total of N times, and each set of values are used to compute values of the five limit state functions G.
Unlike naturally deposited soils, the fill soil in a GRSW is an engineered material and therefore the variabilities in soil unit weight γ_R are low, however the variability of the friction angle φ_R can be as high as 10 % according to a low coefficient of

variation. For the back soil and the foundation soil the variabilities in soil unit weights (γ_s, γ_f) and friction angles (φ_s, φ_f) can be very large. The direct shear factor f_{ds} and the pullout factor f_{po} are usually obtained from direct shear and pullout laboratory tests on the specific geosynthetic and standard and/or site-specific soil. Their value can be set with no variability or with an associated variability, in which case also f_{ds} and f_{po} become stochastic parameters in Monte Carlo simulations. In any case the maximum value of f_{ds} and f_{po} shall be 1.0, since the interface cannot afford higher friction angle than the adjacent soil.

All the stochastic parameters are assumed to vary, according to the set variabilities, with higher or lower values than the nominal value, except for the friction angle of foundation soil φ_f, which can assume only lower values than the nominal value (otherwise the resulting extremely large variability of N_γ can make the standard deviation of G_{bc} larger than its average value).

The probability distribution of all parameters is assumed to be the normal distribution.

For GRSW the length L and the ultimate tensile strength of geogrids T_{ult} are the output of the design, since usually the vertical spacing is given by the facing or the geometry and it is not considered as stochastic.

From the N Monte Carlo iterations the average value and standard deviation of the five limit state functions G is calculated; then the Excel function NORM.INV allows to calculate the value of each G function corresponding to 0.1 % probability of failure (that is, corresponding to 0.1 % probability that $G \leq 0$), which is assumed to be a reasonable engineering limit for GRSW, as above discussed.

Hence the length L or the ultimate tensile strength of geogrids T_{ult} are varied by trials and errors for each limit state until the G value with 0.1 % probability becomes approx. equal to 0.

Following this procedure the values of L and T_{ult} producing 0.1 % probability of failure for each of the five limit states can be evaluated, together with the corresponding average values of the five limit state functions G. Obviously the probability of failure can be set either larger or smaller than 0.1 %.

The above framework remains valid even when other parameters are considered as stochastic (as example, the direct shear factor f_{ds}, the pullout factor f_{po}, and the surcharge q can be considered with an associated percent variability): it is enough to introduce their stochastic values in the limit state equations when producing the N Monte Carlo iterations.

Moreover, the above framework is based on the "traditional" method of calculating the limit state equations with the nominal values of parameters; yet, the framework remains valid even when semi-probabilistic methods are used (like LFRD in USA and EuroCode 7 in Europe) by applying amplification factors to loads and reduction factors to resistances: in this case it is enough to substitute the values of q, φ, γ, etc., with the factorized values in Equations (2) ÷ (15).

5. EXAMPLE OF RELIABILITY ANALYSIS OF GRSW

Acknowledging the use of geosynthetics, characterized by a low coefficient of variation compared to geotechnical parameters, will lead to a safer and efficient design. In order to illustrate the influence of the geosynthetics in a soil reinforcement system, the 10 m high wall shown in Fig. 4 is considered, with the input data in Table 1, which shows also the assumed variabilities of parameters. When variability is zero the parameter is considered as non-stochastic. For sake of simplicity, only cohensionless soils are considered in this example. The reinforcement is assumed to be bonded polyester geogrids, for which typical values of RFs, f_{ds}, and f_{po}, are assumed (BBA, 2010). The vertical spacing between the reinforcements is set as 0.6 m. Surcharge is a uniform load applied along the top horizontal surface of the wall with a set value of 20 kPa.

The first analysis is aimed to calculate the tensile strength in the reinforcement, which depends on the properties of the fill. In this example, the cohesion was omitted to reduce the calculation as it is good practice to consider the fill as cohesionless.

In this analysis, multiple parameters are modeled as random variables (the ones with variability > 0 in Tab. 1), including geotechnical and geogrid parameters. The nominal values and percent variabilities of these parameters for one specific run of the Monte Carlo simulations are summarized in Table 1. The variability for the friction angle (φ_r) and unit weight (γ_r) of reinforced fill is varied between 5 % and 20 % (corresponding to reasonable variabilities in case of very good to poor controls on the fill), and from 5 % to 40 % for the friction angle (φ_s, φ_f) and unit weight (γ_s, γ_f) of back soil and foundation soil (corresponding to reasonable variabilities in case of very good to very poor geotechnical investigations). The percent variability for the ultimate tensile strength T_{ult} and the Reduction Factors of the reinforcement are assumed as 1 %, considering the chemical and biological degradation, installation damage and creep effects. Geometrical and load parameters are assumed with no variability.

A probabilistic analysis using a Monte Carlo simulation with 10.000 iterations was applied to the model considering the variation of the stochastic parameters, as shown in Table 1 for a specific run. The Monte Carlo simulation is used as a mean to compute the probability of failure and the values of length and tensile strength of geogrids associated with the probability of failure of 0.1 %, for a given set of combination of the values of the stochastic parameters.

The Monte Carlo simulations can be used for evaluating the probability of failure associated with a given variability of one stochastic parameter while all other parameters are set to given nominal values.

Table 1. Input data for one run of Monte Carlo simulation for the example calculation

WALL GEOMETRY	SYMBOL	VALUE	UNIT	VARIABILITY (%)
Height	H	10,00	m	0,00
Length of geogrids	L	5,60	m	0,00
Vertical spacing of geogrids	S	0,60	deg	0,00
Number of geogrids	Ngg	17,00	-	0,00
Uniform surcharge	q	20,00	kPa	0,00
SOIL PARAMETERS	**SYMBOL**	**VALUE**	**UNIT**	**VARIABILITY (%)**
Saturated unit weight of the reinforced soil	γ_R	18,00	kN/m^3	5,00
Friction angle of the reinforced soil	φ_R	32,00	deg	5,00
Cohesion ofthe reinforced soil	c'_R	0,00	kPa	20,00
Saturated unit weight of the backfill soil	γ_S	18,00	kN/m^3	20,00
Friction angle of the backfill soil	φ_S	30,00	deg	20,00
Cohesion of the backfill soil	c'_S	0,00	kPa	20,00
Saturated unit weight of the foundation soil	γ_f	18,00	kN/m^3	20,00
Friction angle of the foundation soil	φ_f	28,00	deg	20,00
Cohesion of the foundation soil	c'_f	0,00	kPa	20,00
INTERFACE PARAMETERS	**SYMBOL**	**VALUE**	**UNIT**	**VARIABILITY (%)**
Direct shear factor	f_{ds}	0,95	m	0,00
Pullout factor	f_{po}	0,95	m	0,00
GEOSYNTHETIC REINFORCEMENT	**SYMBOL**	**VALUE**	**UNIT**	**VARIABILITY (%)**
Description of Geosynthetic reinforcement	Bonded polyester geogrid			
Tult of geogrids	Tult	70,00	kN/m	1,00
Reduction Factor for creep	RF_{cr}	1,39	-	1,00
Reduction Factor for installation damage	RF_{ID}	1,10	-	1,00
Reduction Factor for chemical damage	RF_{ch}	1,20	-	1,00
Reduction Factor for biological damage	RF_b	1,00	-	0,00

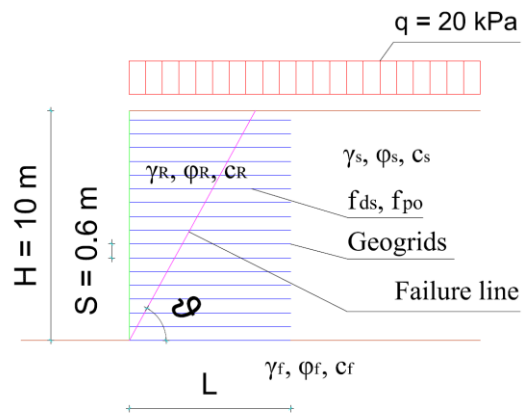

Figure 4. GRSW for the example calculation

As example, if we consider only the friction angle of back soil φ_s as a stochastic parameter and we set all other parameters with the nominal values in Tab. 1, the probability of failure P_f for sliding along the base varies from 0.17 % to 21.68 % when the variability, expressed as coefficient of variation CV (%), of the friction angle of back soil varies from 10 % to 40 % from its nominal value in Tab. 1 (Fig 5). In terms of reliability β, referring to Figure 1, the design moves from above average which has a reliability index of 2.94, very close to the SANS 10160-5 requirements, to an index of 0.79 which is a very hazardous design. These results would suggest to increase the number of geotechnical tests on the back soil for reducing the variability of the nominal value of φ_s to less than 10 %.

Using the same procedure, if we consider only the friction angle of foundation soil φ_f as a stochastic parameter and we set all other parameters with the nominal values in Tab. 1, the probability of failure for sliding along the base varies with $CV(\varphi_f)$ as shown in Fig. 6: an increase in the variability of the friction angle below the wall increases the probability of failure increases from 15 % to more than 45 %. These results would suggest that with the set length L of 5.60 m (Tab . 1) it would be impossible to reach a value of P_f of 0.1 %. Hence the length would first be increased and this analysis would be repeated.

The Monte Carlo simulations can be used to evaluate the minimum value of one parameter to keep the probability of failure at less than 0.1 % when other stochastic parameters varies with given variabilities.

As example, a further analysis varied the tensile strength of the geogrid in order to keep the probability of failure P_f at less than 0.1 % when $CV(\varphi_R)$ varies from 5 % to 20 %, and while $CV(\varphi_b) = CV(\varphi_f) = 20$ %. From the results in Figure 7, the tensile strength increases with the increase of $CV(\varphi_R)$ from 68 kN/m to 90 kN/m. Such analysis would suggest that increasing the controls on the fill, in order to decrease $CV(\varphi_R)$, would allow to reduce the required tensile strength of the geogrid. Then the cost of increased controls could be compared with the decreased cost of geogrids.

While the tensile strength only affects the stability in terms of tensile rupture, the length of the geosynthetics influences all the other limit state functions.

The same approach can then be followed considering the variation of the foundation properties and the effect on the length of the geogrid: the minimum length L to keep the probability of failure P_f at less than 0.1 %, considering all limit state functions, when $CV(\varphi_b) = CV(\varphi_f)$ varies from 5 % to 40 %, while $CV(\varphi_R) = 5$ %, was investigated and the results are shown in Fig. 8, where it can be seen that the minimum length varies from 5.87 m to 13.0 m. These results could be used to compare the increased cost of geotechnical investigations, required to keep $CV(\varphi_b) = CV(\varphi_f) = 5$ %, with the reduced cost of the reinforced soil wall when the length L can be reduced to 5.87 m.

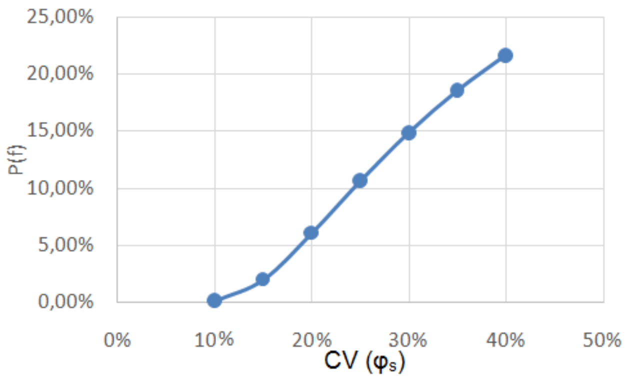

Figure 5. Variation of P(f) for sliding along the base with CV (φ_s)

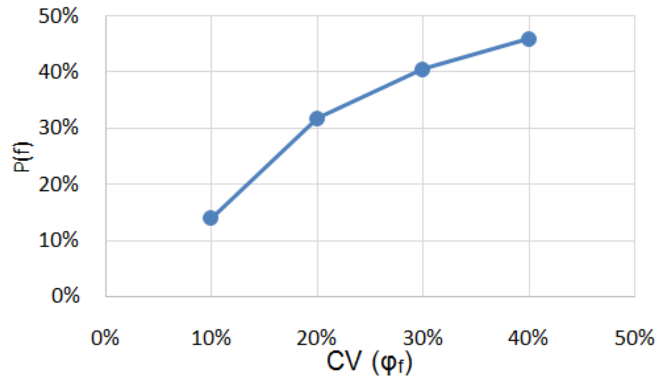

Figure 6: Variation of P(f) for sliding along the base with CV (φ_f)

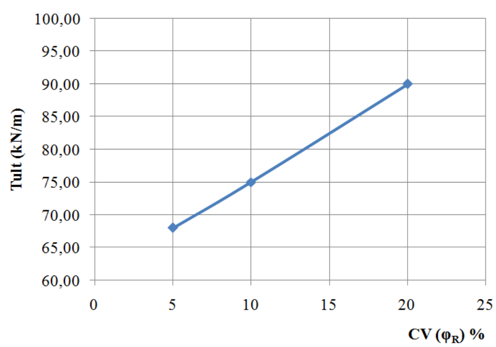

Figure 7. Variation of the tensile strength with CV(φ_R) while CV(φ_s) = CV(φ_f) = 20 %

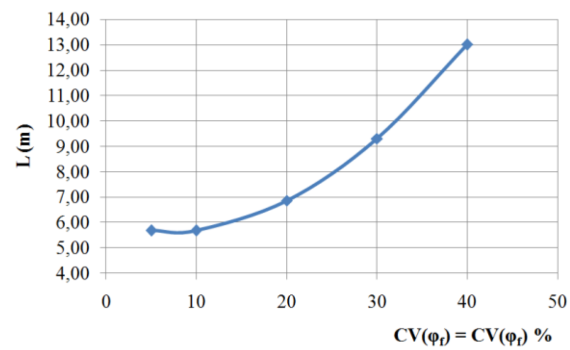

Figure 8. Variation of the length L with CV(φ_s) = CV(φ_f) when CV(φ_R) = 5 %

From all the results of the performed Monte Carlo analyses (few of them are summarized in Figures 5 to 8) the following considerations apply:

1. The Monte Carlo simulation can be very useful for the preliminary design of a RSW, since it affords to get the minimum length and tensile strength of reinforcement associated with the set probability of failure.
2. The probability of failure for sliding along the base increases very quickly when the variability, expressed as coefficient of variation CV (%), of the friction angle of back soil increases (Fig. 5).
3. The length L affects all limit states, excluding the tensile failure: setting the length L according to deterministic stability analyses may lead to the impossibility of getting the required probability of failure (Fig. 6).
4. The critical limit state for defining the length of geogrids L, to get 0.1 % probability of failure (0.1 % fractile value of limit state functions G approx. equal to 0), in this specific case, is pullout in case of CV(φ_b) = CV(φ_f) ≤ 10 %, and direct sliding for higher values of CV(φ_b) = CV(φ_f) while CV(φ_R) = 5 % (corresponding to very good control of fill soil); it is interesting to note that the values of CV(φ_b) and CV(φ_f) affect the critical limit state, even moving it from the bottom of the wall (direct sliding failure) to the top of the wall (pullout failure). This phenomemnon would be impossible to catch with only deterministic calculations.
5. The minimum ultimate tensile strength T_{ult} of the specific geogrids used in this example varies from 68 to 90 kN/m (Fig. 7), for getting 0.1 % probability of failure (0.1 % fractile value of G_{tf} approx. equal to 0), when CV(φ_R) varies from 5 % to 20 %, while CV(φ_b) = CV(φ_f) = 20 %, corresponding to medium quality of geotechnical investigation.
6. The probabilistic analysis can be easily repeated by changing the variabilities and the number of the stochastic parameters: in this way different options can be quickly investigated.
7. As example, the option of increasing the number of lab and in-situ tests to get 5 % variabilities for back soil and foundation soil parameters, would allow comparing the higher cost of testing vs the reduced cost of the GRSW; Fig. 8 reports the result of this exercise: with reduced variabilities of back soil and foundation soil parameters, the critical limit state becomes the pullout, and the minimum length is reduced to 5.68 m.
8. It is also possible to compare the higher cost of very good control of the fill, which would allow to reduce the required tensile strength to 68 kN/m (as shown in Fig. 7), with the reduced cost of the GRSW.
9. The probabilistic analysis can be easily repeated by changing the type of reinforcement and the type of fill, thus allowing the comparison (technical and economical) of different design options.
10. It is evident that this framework for probabilistic analyses of GRSW can be a very valuable tool for preliminary design, since it allows to identify the critical limit states and to set the minimum values of L and T_{ult} which define the 0.1 %

probability of failure (or any other set probability of failure). It also allows to evaluate the increased costs of testing vs reduced costs of the GRSW, or vice versa.

6. CONCLUSIONS

Nowadays the benefit on the use of geosynthetics in soil reinforcement applications is well known.
Geosynthetics provide high tensile strength inside the reinforced soil body, thus increasing not only the Factors of Safety against failure but even the reliability of the design.
Soil parameters can have coefficients of variation up to 40 %, when poor, yet cheap, geotechnical investigations provide insufficient or not reliable information.
On the other hand, the use of geosynthetics, which afford very low coefficient of variation for their technical properties, being manufactured under rigorous quality control testing, allows to get the required low probabilities of failure even when the variability of soil parameters is very high, at a fraction of the cost required for high quality geotechnical investigations.
In fact, considering the cost for a thorough geotechnical investigation, especially in remote areas with lack of resources, machineries and adverse conditions, geosynthetics are able to provide a safe and sound design, even with a high variability of geotechnical properties, ensuring an adequately low probability of failure at lower capital expenses.
Uncertainties in the geotechnical and loading parameters have significant effects on the design of geosynthetic reinforced soil walls (GRSW).
This paper is an attempt to describe a rational reliability theory-based approach for the analysis and preliminary design of GRSW. A reliability analyses framework is introduced for external and internal stability of GRSW, considering the variability of several parameters required in the design process.
An example application is carried out for a GRSW with granular soils to demonstrate the potential of the proposed reliability framework. Results show that the probability of failure decreases with improved reliability of geotechnical data, which in turn requires higher costs for geotechnical investigations.
The proposed framework can be very useful for engineers to make a more informed design decision based on target reliability requirements, considering the costs of geotechnical investigations vs the cost of the GRSW.

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