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Some implications of slope stability analysis methods applicable to heap leach pads

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ABSTRACT

Slope stability analysis is a critical component of geotechnical engineering applied to heap leaching. For decades, analytical techniques and software based on the limit equilibrium method (LEM) have been the basis for slope stability analyses. It is well understood that these techniques do not account for the deformational behavior of the slope materials and many complex failure mechanisms cannot be modeled by the LEM. However, the calculated factors of safety are generally conservative. This positive experience has given the LEM somewhat of a model stature despite its limitations.

A more rigorous method using finite element or finite difference techniques and a process labeled as shear stress reduction (SSR) is gaining some ground. In this method, shear strength is incrementally reduced in the model to induce failure, and the ratio of the actual shear strength to reduced strength at failure is taken as the factor of safety. For slopes with uniform material properties, the SSR and LEM have produced similar factors of safety. However, in cases of slopes with complex profiles including multiple materials, geosynthetic, sharp geometric changes, etc., both techniques can produce anomalous results and reliability of the calculated safety factors becomes questionable. For example, in a complicated slope profile, LEM could result in forces implying movements contrary to intuition and reality. On the other hand, in the SSR process, selective or indiscriminate application of strength reduction to some or all materials could significantly impact the calculated factor of safety. This paper discusses some of these implications.

RESUMEN

El análisis de estabilidad de taludes es un componente crítico de la ingeniería geotécnica aplicada a la lixiviación en pilas. Durante mucho tiempo, las técnicas analíticas y programas de cálculo basados en el método de equilibrio límite (LEM) han sido la base para los análisis de estabilidad de taludes. Es bien sabido que estas técnicas no tienen en cuenta el comportamiento de deformación de los materiales del talud y muchos de los mecanismos de falla no pueden ser modelados usando este método; para compensar esta condición se usan factores de seguridad generalmente conservadores que han producido resultados muy positivos en los diseños, lo cual hace que este método sea reconocido como el preferido a pesar de sus limitaciones.

Hoy en día, un método más riguroso que usa la técnica de elementos finitos o de diferencias finitas y un proceso denominado reducción del esfuerzo cortante (SSR) está siendo cada vez más aceptado. En este método, la resistencia al corte se reduce gradualmente hasta inducir la falla; la proporción entre la resistencia al corte actual y la resistencia reducida al momento de la falla se toma como el factor de seguridad. Para taludes con propiedades de material uniformes, el SSR y el LEM han producido factores de seguridad similares. Sin embargo, en casos de taludes con propiedades o componentes más complejos como aquellos que incluyen varios tipos de materiales, geosintéticos, cambios geométricos significativos, etc., ambas técnicas pueden producir resultados muy diferentes y la confiabilidad de los factores de seguridad calculados por cada método se vuelve cuestionable. En taludes complejos, el método LEM podría resultar en fuerzas que implican movimientos contrarios a la intuición y la realidad; y el método SSR (donde se aplica selectiva o indiscriminadamente la reducción de resistencia de algunos o todos los componentes del talud) podría resultar en factores de seguridad muy diferentes. El documento discute algunas de estas implicaciones.

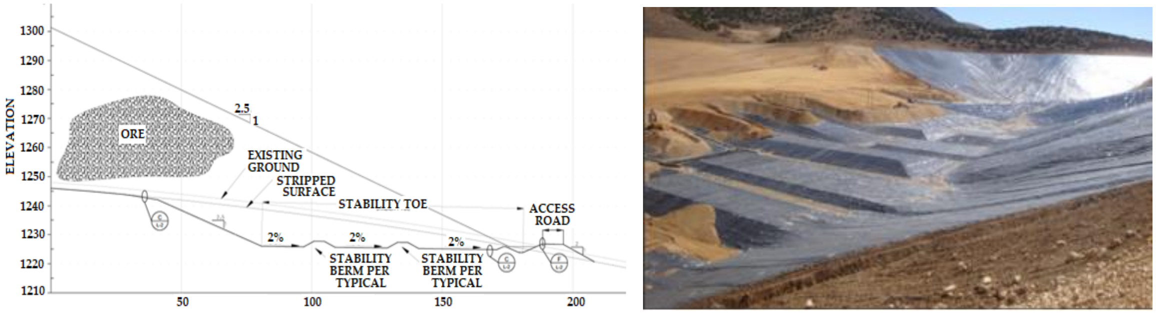
1. INTRODUCTION

The Limit Equilibrium method (LEM) has been the most popular slope stability analysis technique for some decades. Pyke (2017), has given a detailed discourse on the pros and cons of LEM. One of the greatest criticisms of the method is its failure to consider the deformational characteristics of the slope material(s). Laboratory experiments almost half a century ago (Roscoe, 1970) showed that the rupture surface of a failed slope coincided with the highest shear strain zone; that is, the zone of highest deformation. Thus, without due consideration to stress-strain behavior, defining the failure surface becomes a trial & error process. Further, planar to circular and log-spiral failure surfaces, which may or may not represent the critical shear strain zone, are used in the LEM to estimate the minimum factor of safety. Nevertheless, in LEM-based analyses the “recommended factor of safety values for slopes and excavations generally ensure that deformations are within acceptable range”, (Rocscience, 2004).

More recently, there has been a thrust towards using finite element/shear stress reduction methodology (SSR) for analyzing slope stability, (Dawson et al. 1999, 2015; Gover and Hamma, 2013; Hamma et al, 2005). The rigorous numerical procedure of modeling the materials and their stress-strain behaviors in the finite element method is undeniably more accurate than LEM. Other advantages of the SSR technique have also been pointed out by various authors and software developers (Dawson et al. 2000, Farshidfar and Nayeri, 2015). However, because of the long history of reasonably good experience with LEM, there is a tendency to use the LEM factor of safety as a gage for approval and acceptance of SSR analyses, or at a minimum, to apply the same minimum factor of safety standard for acceptance in SSR analyses. That is, one-to-one comparisons are being made between the safety factors obtained by the two techniques without full attention to the idiosyncrasies of each.

2. METHODOLOGY, RESULTS & DISCUSSION

One situation where the shortcoming of the LEM-based analysis is apparent is becoming commonplace in the heap leaching industry. For steep or complex leach pad geometries designers increasingly use stabilizing berms, also referred to as “speed bumps,” to improve the calculated factors of safety. Sometimes these stabilizing berms are simply 1 m high triangular or trapezoid cross sections as shown in Figure 3(a); other times these can form a more complex network of berms as shown in the photograph in Figure 3(b) (Breitenbach and Athanassopoulos, 2013). The underlying idea is that, in this arrangement, the potential failure surface is disrupted and forced to either shear through the berm or ride over it and pass locally through the higher strength ore.



a) Drawing

b) Photograph

Figure 1. Two different heap leach pads showing stability berms

It is not uncommon in LEM-based analyses to consider the failure surface along the liner following, at least in part, the berms up, over and down, unlikely as it may be, considering the sharp features of the berm relative to the size of the failure surface. In the method of slices, this implies that some of the slices must ride up the slope against the gravitational force, while those on the downward slope slide down, aided by the gravity. That is, there is a reversal of the inter-slice shear forces at the berm. However, the calculated inter-slice forces do not support this; for all cases considered by the authors these forces were calculated to be in the same direction and of similar value as for the slices away from the berm. In other words, because the LEM method disregards strains, the analysis does not recognize that there must be a stress direction shift as the ore attempts to ride up the berm’s face (at least for safety factors close to 1.0). Thus, there is some doubt as to the reliability of LEM-calculated factors of safety for sections with stability berms or other sharp geometric features.

In the SSR technique, the shear strength of the material constituting the slope is reduced in steps and a stress-strain relationship-based finite element analysis is carried out at every step to find the limiting state defining the failure. The ratio ($\boldsymbol{\omega}$) of the actual or initial shear strength ($\boldsymbol{\tau}$) to the reduced shear strength at the limiting state ($\boldsymbol{\tau}$’), is labeled as the Critical Shear Reduction Factor (SRF) and considered a measure of the safety factor against slope failure.

Thus, $\boldsymbol{\omega} = \frac{\boldsymbol{\tau}}{\boldsymbol{\tau}'}$ [1]

Rewriting equation (1) for SSR technique as: $\boldsymbol{\tau}' = \boldsymbol{\tau}/\boldsymbol{\omega}$ and using the Mohr-Coulomb strength criterion ($\boldsymbol{\tau} = c + \boldsymbol{\sigma} \tan \phi$ and $\boldsymbol{\tau}' = c' + \boldsymbol{\sigma} \tan \phi'$, where the pairs c~ϕ and c’~ϕ’ are shear strength parameters corresponding to the actual and limiting states, respectively) we get,

$$\mathbf{c}' + \boldsymbol{\sigma}.\mathbf{Tan}\,\boldsymbol{\phi}' = \frac{\mathbf{c}}{\omega} + \sigma.\frac{\mathbf{Tan}\,\boldsymbol{\phi}}{\omega} \qquad [2]$$

implying that,

$$\mathbf{c}' = \frac{\mathbf{c}}{\omega} \quad \& \quad \mathbf{Tan}\,\boldsymbol{\phi}' = \frac{\mathbf{Tan}\,\boldsymbol{\phi}}{\omega}$$

The Factor of Safety (F) in LEM, on the other hand, is taken as the minimum value of the ratio of resisting and driving forces along a potential failure surface. Thus,

$$\mathbf{F} = \frac{\int_0^l (\tau_l)\,\mathrm{dl}}{\int_0^l (\tau_s)\,\mathrm{dl}} = \frac{\int_0^l (c + \sigma.\mathrm{Tan}\,\phi)\mathrm{dl}}{\int_0^l (\tau_s)\,\mathrm{dl}} \qquad [3]$$

where, τ_1 is the shear strength developed and τ_s, the shear stress along the failure surface l; c and ϕ are the shear strength parameters, and σ is the normal stress component across the failure surface.

As noted by Zheng et al. (2009), dividing both sides by F produces the limiting state corresponding to Factor of Safety =1.0:

$$\mathbf{1} = \frac{\int_0^l (c/F + \sigma.\mathrm{Tan}\,\phi/F)\mathrm{dl}}{\int_0^l (\tau_s)\,\mathrm{dl}} = \frac{\int_0^l (c' + \sigma.\mathrm{Tan}\,\phi')\mathrm{dl}}{\int_0^l (\tau_s)\,\mathrm{dl}} \qquad [4]$$

Where, c'= c/F and Tan ϕ'=Tan ϕ/F

In other words, the shear strength or the shear strength parameters in the LEM are reduced by a factor F to produce the limiting state, just as ω does in case of SSR. Thus, one may say that the critical shear stress reduction factor (ω) in SSR technique is equivalent to the safety factor (F) in LEM.

This demonstration of equivalence is reasonable and seems to build some confidence in using the SRF obtained through SSR technique as a direct replacement for LEM factor of safety. Nevertheless, this theoretical equivalence must be viewed in the light of some practical considerations and characteristics of numerical tools, which include:

- the entire slope considered in the above demonstration is comprised of a single uniform material,
- the physical significance of reliance on shear strength reduction process as a means of inducing limiting or failure condition is unclear, and,
- there is no universally accepted or absolute definition of limiting state considered in the strain-based analysis using finite element or any other numerical technique.

Gover and Hamma (2013) compared the results of the LEM and SSR methods for three lined slopes. Slope 1 comprised brine and sludge ponds proposed for an existing coal slurry facility. Slope 2 consisted of a lined tailings facility on a sloping foundation. Slope 3 comprised a tailings impoundment with a multi-layered soil profile below the foundation and an assumed leak in the liner. For all three slopes both LEM and SSR produced similar critical slip surfaces, but there was some inconsistency in the calculated factors of safety, as summarized below:

- Slope 1: $FOS_{LEM} = 1.43$; $FOS_{SSR} = 1.3$
- Slope 2: $FOS_{LEM} = 1.37$; $FOS_{SSR} = 1.2$
- Slope 3: $FOS_{LEM} = 1.46$; $FOS_{SSR} = 1.46$

There are various ways in which a limiting or failure condition can be induced, shear strength reduction being one of them. A stable slope under the normal conditions may reach the limiting condition during an earthquake; Cheng et al. (2007) discuss an alternative of increasing gravity, analogous to how earthquake loading is considered in pseudo-static analyses. Hamma et al. (2005) duplicated the LEM factor of safety results for some simple slopes without any reinforcement by assuming an elastic-perfectly plastic post-peak behavior with the same elastic constants (Young's Modulus and Poisson's Ratio) and zero dilation angle for all materials in SSR analysis. Each of these conditions may represent different deformation patterns, suggesting different failure surface.

The limiting state in the SSR method is commonly identified as a specified maximum strain value anywhere in the model, which can be indirectly referred to by non-convergence of the solution. The specified maximum strain may vary from one program to another or may even be specified by the user, while the convergence depends on the solution technique, convergence criterion, any special algorithm adopted to treat large deformations, discretization characteristic, and a host of other things. Also, some have considered the limiting state reached when large deformations have occurred over a potential failure surface covering the entire slope height, others have treated it as the initiation of instability caused by large deformations even in a small area leading to a progressive failure. According to Matsui and San (1992), credited for the first application of SSR to slope stability analysis, failure of the slope is defined as “the stage when the failure shear strains develop from the toe to the top of the slope”, in a sense restricting the definition of failure to a global failure. In this sense the SSR and LEM are similar, in that the LEM generally requires that all slices in a section reach a FOS of 1.0. But herein lies another advantage of the SSR method: local factors of safety can be calculated, and this can be used to determine the risk of progressive failures.

Other complications arise when the slope comprises multiple materials and especially in case of the presence of a geomembrane liner. Some authors have proposed all material strengths to be reduced in the SSR analysis by the same factor without any consideration to potential failure zone or surface. If the failure surface passes through all the materials in the model, the integral terms in eqn. [4] becomes a ratio of summation of shear resistance mobilized over the failure surface segments through each material and the summation of the corresponding driving shear forces. Thus, a division by F and the implied shear strength reduction applies to all materials and, in some sense the equivalence between LEM and SSR is demonstrated. But, imagine a situation when the critical failure surface as predicted by the LEM does not pass through all the materials. In this case, the integrals in eqn. [4] do not include all materials and it is not clear how, or if, the equivalence of two techniques is established. This is analogous to the findings of Cheng et al. (2007) where they found that SSR methods were incapable of determining failures surfaces, which may be only slightly less critical a shortcoming.

While neither the LEM nor SSR method is meant to be quantitative risk assessments, it is common to use the calculated FOS as an indicator of risk of failure; see, for example, Ching 2009 and Silva et al 2008. This is also implicitly done anytime engineers assign a minimum design FOS which is based on duration of exposure (e.g., 1.30 for construction or temporary slopes versus 1.50 for permanent slopes). It is also common that the geotechnical properties of some materials are less well known than others, or more prone to having lower strengths than assumed. Thus, from a risk assessment perspective, it may sometimes be useful to reduce the shear strength of some layers more than others. However, in the finite element analysis based SSR technique there is no prior indication whether the failure surface will pass through one, more than one, or all materials and there is no way to force the failure surface, as we intuitively or explicitly do in case of LEM. Thus, it becomes difficult to figure out if one, some, or all material strengths should be reduced. That said, some have considered selective application of SSR to materials and the liner interfaces, as in commercial software *RS*^{2} (Rocscience, 2018). The impact of selective or indiscriminate shear strength reduction on the calculated deformation pattern or the potential failure zone as well as the resulting critical SRF is not understood, nor has been addressed in the literature. Most recent efforts have been focused mostly on developing SSR-based software and on equating the numerical values of SRF from SSR to FOS from LEM and, the inferences thereof. This approach leaves the validity of universal application of SSR technique a bit murky.

The following examples illustrate the effects of, and the ambiguity presented by the selective or indiscriminate way SSR is applied. Example 1 considers a simple slope comprised entirely of sand fill deposited over a soft cay layer. The slope dimensions and the material properties assumed in the analysis are shown in Figure 2. Three cases were analyzed with shear strength reduction applied to both soft clay and sand layers. Case a) applies the same reduction factor to both layers, case b) applies the reduction only to the clay, and case c) only to the sand. The *RS*^{2} software was used for the analyses. The critical SRFs obtained were 1.26, 1.34 and 1.45, respectively, for the cases -a), -b) and -c). The obvious question arises: which one to choose and on what basis? The maximum shear strain patterns for the limiting states in cases a) and b) are practically identical, suggesting failure surfaces of a global nature touching the model bottom and passing through both layers. The same cannot be said with any level of certainty for case c) with shear strength reduced in the upper soft clay layer only, due to an additional, high shear strain pocket developed near the slope crest. Once again, the question arises, what makes us elect one or the other as the design factor of safety: one as recommended by some – case a), one with the maximum shear strain – case b), or one suggesting a potential local failure – case c)?

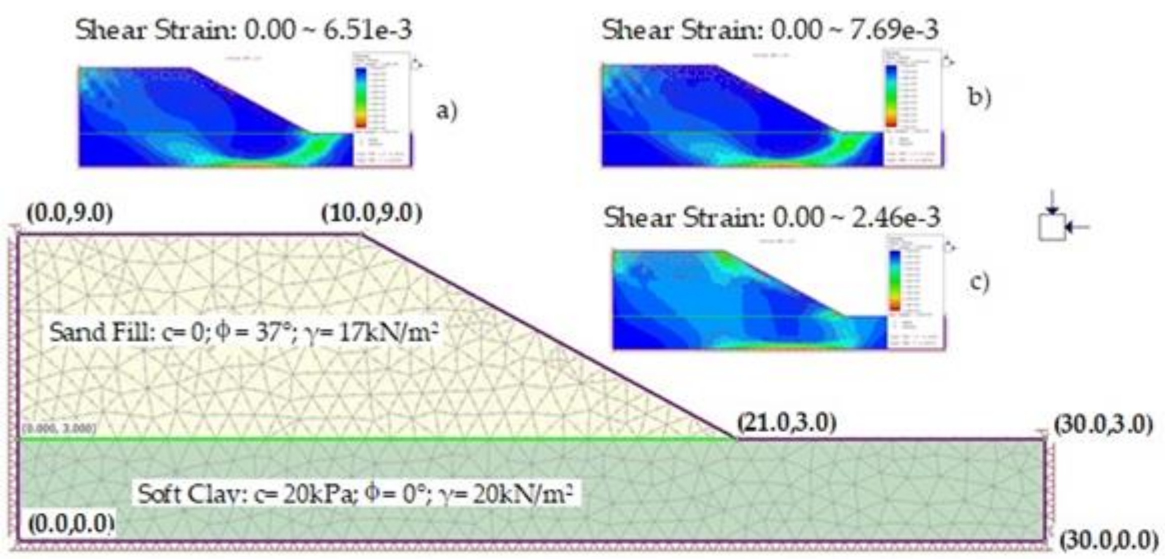
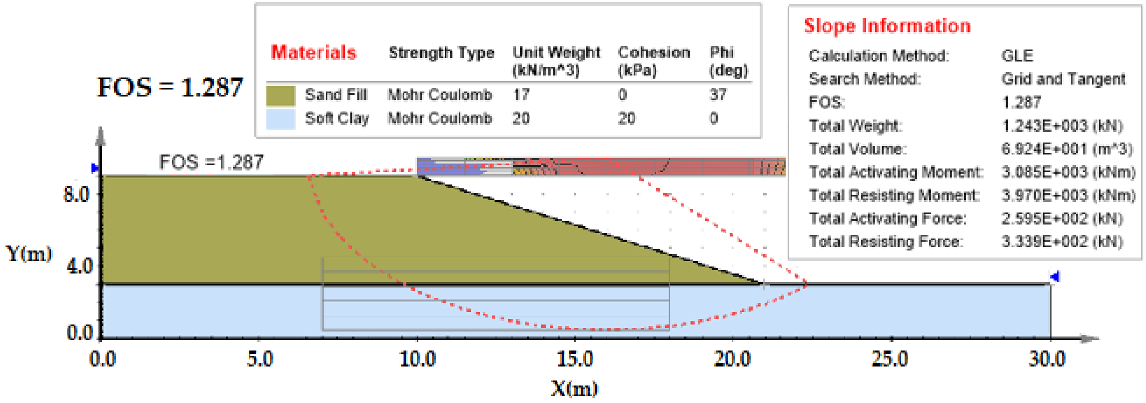


Figure 2. SSR-based stability analysis of a simple slope with sand fill deposited on soft clay; SSR applied to (a) both layers, (b) soft clay layer only, (c) sand fill only.

Figure 3 shows the result of a LEM-based stability analysis of the same slope using another commercial software – SVSLOPE (Soilvision). The factor of safety considering a circular failure surface is calculated as 1.29 and the critical failure surface is of global nature conforming to the critical SRF of 1.26 and the maximum shear strain zone suggested in the SSR analysis with shear strengths reduced for both layers. One may surmise that global failures being of prime concern, this settles the case in favor of an indiscriminate shear strength reduction to all slope materials and restores the confidence in SSR-based analysis. However, if the slope becomes a bit more complicated, this may not be true.



Materials	Strength Type	Unit Weight (kN/m^3)	Cohesion (kPa)	Phi (deg)
Sand Fill	Mohr Coulomb	17	0	37
Soft Clay	Mohr Coulomb	20	20	0

Slope Information

Calculation Method:	GLE
Search Method:	Grid and Tangent
FOS:	1.287
Total Weight:	1.243E+003 (kN)
Total Volume:	6.924E+001 (m^3)
Total Activating Moment:	3.085E+003 (kNm)
Total Resisting Moment:	3.970E+003 (kNm)
Total Activating Force:	2.595E+002 (kN)
Total Resisting Force:	3.339E+002 (kN)

Figure 3. Conventional LEM-based stability analysis of the slope with sand fill deposited on soft clay

For example, consider the same slope as described in the above example, with the sand fill and the soft clay layers, but separated by a geomembrane liner. Figure 4 represents the SSR-based analyses of this slope, with the liner modeled as a geogrid sandwiched between two joints representing the interfaces - a modeling option provided in RS^2 software. The properties of the sand fill and soft clay layers remain the same as used in the previous example (Figure 1) and that of the liner elements are indicated in Figure 4.

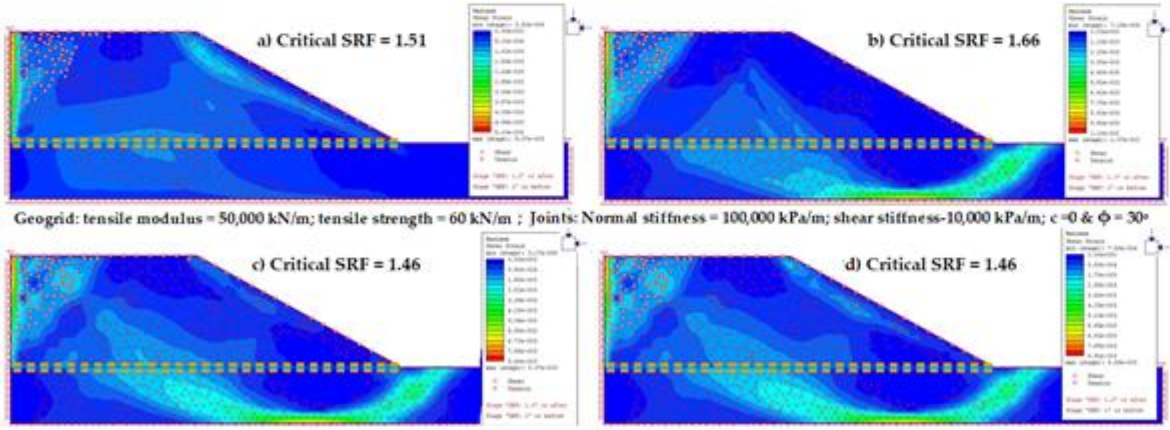
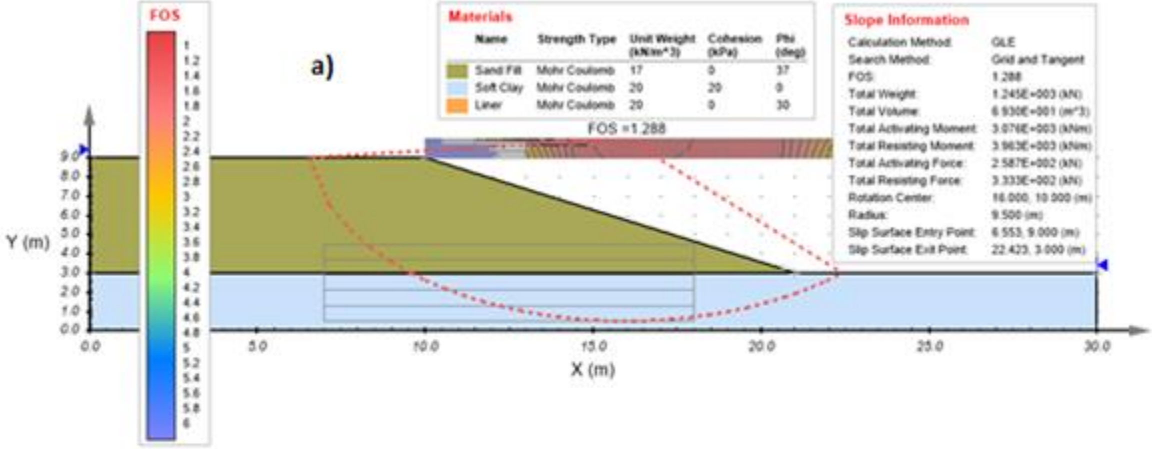


Figure 4. SSR-based analysis of the slope with sand fill deposited on soft clay with a liner in between; SSR applied to a) sand and liner only b) clay and liner only c) sand and clay only d) all- sand, clay and liner

The model and the properties are duplicated from a tutorial problem in RS^2 User's Manual. Four cases were analyzed with shear stress reduction applied to any two or all three slope components – sand, clay and the liner. Since the geogrid is taken as the tensile element only, the shear strength reduction to the liner is applied only through the interfaces represented by the joints in the model. The critical SRF obtained in the four cases were 1.46 to 1.66, with 1.46 being reported in the RS^2 tutorial. The SRF obtained through various SSR application strategies vary only by 0.2, but with the design criteria being FOS of 1.5 in many cases, this would make the difference between the design being implemented or rejected.

LEM analysis of this model using SVSLOPE in which the geomembrane is modeled as a very thin layer attributed with the same shear strength characteristics as that assigned to the interfaces in the SSR analysis provides a FOS of 1.29 associated with a circular slip surface as shown in Figure 5 and 1.83 for a global block failure along the liner and passing through the slope crest.

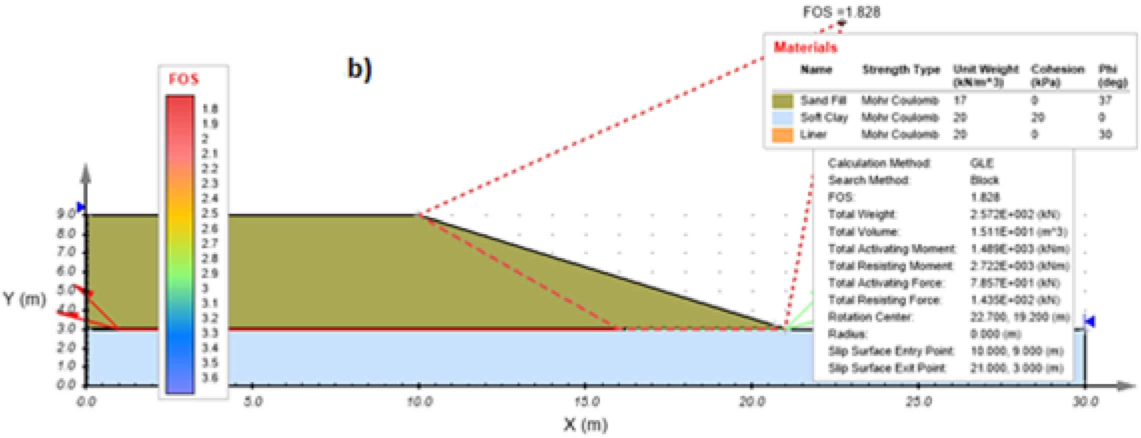


Materials

	Name	Strength Type	Unit Weight (kN/m^3)	Cohesion (kPa)	Phi (deg)
🟫	Sand Fill	Mohr Coulomb	17	0	37
🟦	Soft Clay	Mohr Coulomb	20	20	0
🟧	Liner	Mohr Coulomb	20	0	30

Slope Information

Calculation Method:	GLE
Search Method:	Grid and Tangent
FOS:	1.288
Total Weight:	1.245E+003 (kN)
Total Volume:	6.930E+001 (m^3)
Total Activating Moment:	3.076E+003 (kNm)
Total Resisting Moment:	3.963E+003 (kNm)
Total Activating Force:	2.587E+002 (kN)
Total Resisting Force:	3.333E+002 (kN)
Rotation Center:	16.000, 10.000 (m)
Radius:	9.500 (m)
Slip Surface Entry Point:	6.553, 9.000 (m)
Slip Surface Exit Point:	22.423, 3.000 (m)



Materials

	Name	Strength Type	Unit Weight (kN/m^3)	Cohesion (kPa)	Phi (deg)
🟩	Sand Fill	Mohr Coulomb	17	0	37
🟦	Soft Clay	Mohr Coulomb	20	20	0
🟧	Liner	Mohr Coulomb	20	0	30

Calculation Method:	GLE
Search Method:	Block
FOS:	1.828
Total Weight:	2.572E+002 (kN)
Total Volume:	1.511E+001 (m^3)
Total Activating Moment:	1.489E+003 (kNm)
Total Resisting Moment:	2.722E+003 (kNm)
Total Activating Force:	7.857E+001 (kN)
Total Resisting Force:	1.435E+002 (kN)
Rotation Center:	22.700, 19.200 (m)
Radius:	0.000 (m)
Slip Surface Entry Point:	10.000, 9.000 (m)
Slip Surface Exit Point:	21.000, 3.000 (m)

Figure 5. LEM analysis of the slope with sand fill deposited on soft clay with a liner in between;
a) circular failure specified b) global block failure with sliding along the liner.

It is clear that the examples shown here in this paper do not show or testify to an unqualified equivalence of the results of slope stability analysis by the two methods under consideration and replacement of the long-used Limit Equilibrium method by shear strength reduction-based finite element technique without further research is not conceivable.

3. CONCLUSION

Limit Equilibrium method is a well-established and so far, the most widely used technique for slope stability analysis despite its limitations in the theoretical arena, especially its lack of consideration of the deformational characteristics of the slope materials. The continued use and popularity of the technique in conjunction with the prevailing limiting factor of safety practice, lies to a great extent, on the positive experience of producing safe slopes, rather than the correctness and rigor of the approach.

More recently, shear strength reduction method utilizing finite element technique is being forcefully promoted. Undoubtedly, the finite element technique affords a rigorous treatment of the deformational characteristics of the slope materials and affords many other possibilities, for example, consideration of progressive failure of slopes. However, the physical significance of the shear strength reduction process, beyond a simple slope of a single material with uniform properties, and the implications of selective or indiscriminate strength reduction are not entirely understood, nor is there a unified effort to develop more clarity in this regard. Instead, the focus seems to be on capitalizing on the acceptance of limit equilibrium method and the associated limiting factor of safety design approach and through case analysis, simply demonstrate that the shear strength reduction method yields similar factor of safety. Such an emphasis dilutes the rigor of finite element technique and introduces a heuristic element to the shear strength reduction method; it is fair to say that a clear understanding of the physical process of strength reduction, is warranted.

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